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THE DISTRIBUTIONAL EFFECTS OF ILLNESS WHEN RECUPERATION IS ENDOGENOUS

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PREFACE

This paper explores the interactions of income and illness through labour supply choices, and the implications for health and health-cost measurements. Marie Moland Gaarder, Ph.D. Economist from University College London, UK, and Consultant in the Social Programs Division II (RE2/SO2), Inter-American Development Bank, is the author of this document.

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Abstract

This paper explores the possible interactions of income and illness through labour supply choices, and the implications for health and health-cost measurements. Standard utility maximisation theory is used to analyse labour supply behaviour under constraints imposed by sickness and minimum consumption requirements. For a rather general utility function, an empirically supported elasticity of substitution between leisure and consumption, and under the assumption that no sick pay is received, we find that only higher-wage individuals will choose to recuperate fully, whereas others will work while being sick as long as this yields additional efficient labour hours. Therefore, the income and welfare losses due to a given illness may be larger for low-wage individuals than for those with higher wages. This paper shows that the traditional models and measurements of illness can lead to distorted estimates of illness by neglecting the interaction between income and health.

Abbreviations

ACWL	Absolute Comparable Welfare Loss
AIL	Absolute Income Loss
C-D	Cobb-Douglas
ENDOR	Endogenous Recuperation Model
ETL	Efficient Time Loss
EXOR	Exogenous Recuperation Model
RCWL	Relative Comparable Welfare Loss
RIL	Relative Income loss
WA	Work Absence

Expressions Guide

EXPRESSION	EXPLANATION
1. Actual recuperation time	The fraction of the *full recuperation time* (see 5) that is actually spent recuperating (at home).
2. Efficient time	Full productivity time-equivalent (i.e. time available multiplied by the level of productivity).
3. Efficient time opportunity frontier/budget constraint	A line showing all combinations of efficient time (see 2) available to an individual depending on his or her choice of how to spend that time (here; in work or leisure/at home).
4. Elasticity of substitution (between consumption and leisure)	Measuring the degree of substitutability between any pair of factors (here; consumption and leisure) or 'the ease with which the varying factor can be substituted for others'. More mathematically expressed: the percentage change in factor proportions due to a change in marginal rate of technical substitution.
5. (Full) recuperation time	The fraction of time it takes to recuperate, under the assumption that the person spends the entire recuperation time at home
6. Health capital	Health capital is a component of the stock of human capital. Factors used in the generation of income, and in which one can invest and which tend to depreciate over time, are generally known as 'capital'. A person's health typically fulfills these criteria. Health capital differs from other forms of human capital in that a person's stock of knowledge affects productivity, whereas his or her stock of health determines both productivity and the total amount of time available to be spent on producing commodities and earnings.
7. Healthy time	Time during which a person is fully productive.
8. Illness severity	Productivity loss during illness in terms of consumption/income produced while working or utility produced during leisure (staying home). The lower the productivity ratio the more severe the illness.
9. Minimum consumption requirement	The amount of consumption (e.g. food, clothing, shelter), expressed in terms of its market price, required to survive.
10. Net minimum consumption requirement	The minimum consumption constraint (see 9) on paid labour
11. Post-recuperation time	The fraction of a certain time period that follows the recuperation time (see 5).
12. Sickness time	Time during which an individual is less than fully productive.
13. Unearned income	An individual's income derived from sources other than employment, such as interest and dividends from investments, or income from rental property.
14. Work-absence	In this paper work-absence (WA) is defined as absence from work during which the individual is sick, and absence is measured as the amount of labour supplied during the illness subtracted from the amount the person would have been working during that period had he or she been healthy.

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1. Introduction

Illness reduces market and non-market productivity, the total amount of time available for production, as well as individual well-being. To avoid or minimise these unwelcome experiences, individuals can invest in their health in order to produce health, or at least restore part of it after an illness. These ideas were presented in an article by Selma Mushkin (1962), and together with Becker (1964) she introduced the concept of health capital, a component of the stock of human capital. The ideas were formalised by Michael Grossman (1972), who constructed a model of the demand for health capital.[1] The health-capital literature has grown apace since Grossman's seminal paper.[2] Whereas Michael Grossman and his followers use sickness time to investigate the demand for health and health care, O'Donnell (1995) investigates the effects of sickness time on household time allocation and consumption decisions. This paper aims to close a gap in the literature on sickness time, with important income and welfare distributional implications, as well as serious implications for commonly used illness-measurement. In particular, the paper adds to the literature on sickness time in three interrelated ways.

The first, and most fundamental extension is to allow for the fact that allocation of sickness time to various activities (e.g. for the purposes of working or recuperation/leisure) may affect the *current* sickness experience, as well as the income and welfare losses incurred from illness. In other words, it recognises that illness is a matter of degree and its impact on daily activities to a certain extent subjective. The theoretical literature has to date treated sickness time as time lost. Grossman (1972) assumes that the uses of healthy time in different activities are mutually exclusive and add up to total time available minus sick time.[3] In particular, the health-capital literature has modelled sick time as a function of health capital, which in turn is affected by investments in health (health care, medicines etc.) and depreciation of health due to factors such as use, age, disease, and pollution. Individuals invest in health capital for consumption and investment reasons.[4] The amount of health investment and thus sick time experienced by an individual will depend on such factors as education and wage rate. However, this literature does not explore how the usage of sickness time (e.g. for the purposes of working, leisure-activities, or seeking medical care) affects *current* sickness, both its duration and severity. In other words, sickness time is endogenous until it occurs – it then becomes exogenous. In contrast to the health capital tradition, which is concerned with the allocation of healthy time and commodities and how these affect the amount of sick time experienced, O'Donnell's model is analyzing how a certain amount of sick time affects household's allocation of time and consumption. Sick time in O'Donnell's model is exogenous, as it does not allow the sickness period to be affected by the household's allocation. In this paper, the conditions under which an individual may decide to work rather than stay at home during the recuperation time from illness will be examined, and the main insights will be the income and welfare distributional implications of assuming endogenous recuperation. Two main and connected research

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[1] Grossman argued that health capital differs from other forms of human capital in that a person's stock of knowledge affects productivity, whereas his or her stock of health determines the total amount of time available to be spent on producing commodities and earnings (Grossman (2000)).)

[2] See for example Cropper (1977), Muurinen (1982), Wagstaff (1986), Selden (1993), O'Donnell (1995), Liljas (1998). A useful survey is provided by Grossman (2000).

[3] In a paper generalising the Grossman model, Muurinen (1982) points out that it is unrealistic to assume that individuals do not use medical care and invest in health during sickness, however, she does not explore the possibility and implications of an individual working during sickness-time.

[4] As a consumption commodity, health may directly enter individuals' preference functions, and as an investment commodity it determines the total amount of time available for market and non-market activities.

questions are addressed: Does wage level affect labour supply behavior under illness? And if so, how does this affect the path of the illness and thereby income and welfare?

An additional insight that results from allowing sickness time to be endogenous is the fact that commonly-used statistical measurements of illness effects, such as work loss days, lead to distorted estimates of illness by neglecting the interaction between income and health. This is the second extension to the sickness time literature.

While the aim is primarily to extend the literature on the endogeneity of sickness time, a crucial side-effect has been the development of a particularly useful way of modelling illness, which is thus the final extension on existing health capital literature made in this paper. Since neither the health-capital literature nor the model developed by O'Donnell allow sickness time to be used either for leisure or work, and because only healthy time is assumed to provide any utility, the severity of a sickness has had no role to play in their models.[5] This paper, however, introduces a way of describing illness that is particularly useful for the case where the recuperation period is endogenous. Sickness is identified by three parameters: duration, severity in terms of productivity loss, and efficiency of recuperation.

The plan of the paper is as follows. Section 2.1 presents the main ideas, assumptions, and definitions of the endogenous recuperation model (ENDOR). Section 2.2 derives the efficient time opportunity frontier for the case of a severe illness case and a less severe illness case, and analyses it with respect to the choices of leisure-time. Section 2.3 uses utility maximisation to explore the relationship between leisure and wage. Section 3.1 explores the implications of the ENDOR model for work-absence, a traditional statistical measurement of illness, whereas section 3.2 looks at the model implications for income loss, and section 3.3 for welfare loss. Section 4 discusses some of the main model limitations, and section 5 concludes and presents some policy implications.

2. The Model

2.1 Basic Model Assumptions and Definitions

An individual may be either sick or healthy, and a period of time for this individual may correspondingly be divided into sick time and healthy time. Healthy time can be thought of as the time in which a person performs all activities at his or her highest productivity level, and derives the most utility out of leisure time. Sickness time, on the other hand, will be identified by lower productivity and lower utility derived from leisure than in the healthy situation.

Sickness will be identified by three parameters: the duration of the sickness, the severity of it, and, finally, the efficiency of the recuperation time. The choice of labour supply will determine the duration of the sickness period. If the person recuperates fully, then the duration of the sickness will be the same as the full recuperation time. If, however, the individual should choose to work part of the full recuperation time (i.e. the actual recuperation time is shorter than full recuperation time), then this will prolong the duration of sickness time (low-productivity time). Severity will be modelled in terms of productivity loss in both work and leisure. The idea behind recuperation time is that some uses of time will be more beneficial for recovery from an illness than others. In particular,

[5] In contrast to the theoretical tradition, the empirical literature has to a certain extent captured the idea that sickness time can be a time of choice (Thomas (1980)[5], Bartel and Taubman (1979)[5], Paul Fenn (1981), Aronsson et al. (2000)). Nevertheless, none of these empirical investigations have analysed how that choice can affect the duration of sickness time.

we are in this model assuming that staying at home (i.e. away from work) will speed up the process of recovery. The efficiency will therefore depend on the time actually spent recuperating, as well as on the severity of the illness. The relationship between severity, duration, and productivity of recuperation will typically vary for different types of illnesses, and for different types of work.

The model uses standard utility maximization theory to analyze an individual's labour supply behavior under constraints imposed by sickness and minimum consumption requirements. We assume that there is no sick pay and that the wage rate depends on productivity. Departing from the previous literature, productivity in work and utility derived from leisure (productivity of leisure time) in this model are influenced by the labour supply decision: if the individual works during exogenously imposed recuperation time, productivity in the post-recuperation time suffers.

We are assuming that there is 1 unit of time, and that any illness will occur at the beginning of this one-unit time-period. In other words, a purely exogenous probability of falling ill is considered. Although the health shock is not affected by individual actions, the development of the illness is. The reason for modelling it this way is two-fold: first of all it may be an acceptable approximation for certain types of health shocks (e.g. an influenza-epidemic, or for the health effects of an increase in air pollution). More importantly, the model is intended to complement the health-as-capital models that show that the level of health capital may vary with income; the current model shows that even for the same amount of potential illness experienced (and implicitly the same amount of health capital) low income individuals may experience higher welfare and income losses than higher income individuals due to the decisions they are forced to make.

If a person falls sick it takes a fraction of the time r ($0 < r < 1$) to recuperate, given that the person spends the entire recuperation time at home. The variable r will be known as *full recuperation time*. We further assume a positive and identical net minimum consumption requirement on income for all individuals, i.e. the minimum consumption requirement is larger than non-wage income by a constant amount.[6] The individual has to decide how much of r to stay at home and how much to work, knowing that working during the recuperation time will prolong the period in which the person is sick. Let x, $0 \leq x \leq 1$, be the fraction of r that is actually spent recuperating (here: staying at home). The variable x will be known as *actual recuperation time*. Assume that the illness reduces the value of time in terms of productivity in work and leisure. Work-time in the recuperation phase (i.e. $r(1 - x)$) has productivity level s, and each hour of r that is spent recuperating is regarded as equivalent to s, $0 < s < 1$, hours of leisure.[7]

The rest of the time, $1 - r$, can also be spent in leisure or work. The effectiveness of post-illness time is affected by the choice of x, i.e. labour supply behaviour during recuperation time. Let z, $0 \leq z \leq 1$, be the fraction of the post-recuperation time spent in leisure (i.e. $z(1 - r)$), and let each hour of leisure be equivalent to $\sigma(x)$, $0 < \sigma(x) \leq 1$, hours of efficient leisure and each hour of work also have productivity $\sigma(x)$. Assume that if the entire recuperation phase is spent recuperating then the post-recuperation phase gives full productivity in work or leisure, i.e. $\sigma(1) = 1$, whereas if the full recuperation time is spent working the productivity in the post-recuperation time will be the same as

[6] Either unearned income and the minimum consumption requirement are the same for all individuals, or if one is higher for an individual then the other one is higher by the same amount.

[7] The assumption of equal productivity in work and leisure is not very important for the results obtained, and simplifies the exposition of the model. As long as the ratio of the productivity of work to the utility derived from leisure stays the same in the recuperation and the post-recuperation phase, the same results will be obtained. To see this refer to appendix 3 B.

during recuperation, i.e. $\sigma(0) = s$. This is another way of stating that if an individual does not take any time to recuperate, the sickness will in effect last the entire period.

For any given choice of x and z the consumer will get efficient leisure:

$$L = s\,x\,r + \sigma(x)\,z\,(1 - r) \qquad (1)$$

This takes the minimum value $L = 0$ when $x = z = 0$, and a maximum value $L = (1\text{-}r)+sr$, when full leisure is taken and $\sigma(1) = 1$. Corollary to this choice of x and z is the amount of efficient work:

$$H = s\,(1 - x)\,r + \sigma(x)\,(1 - z)\,(1 - r) \qquad (2)$$

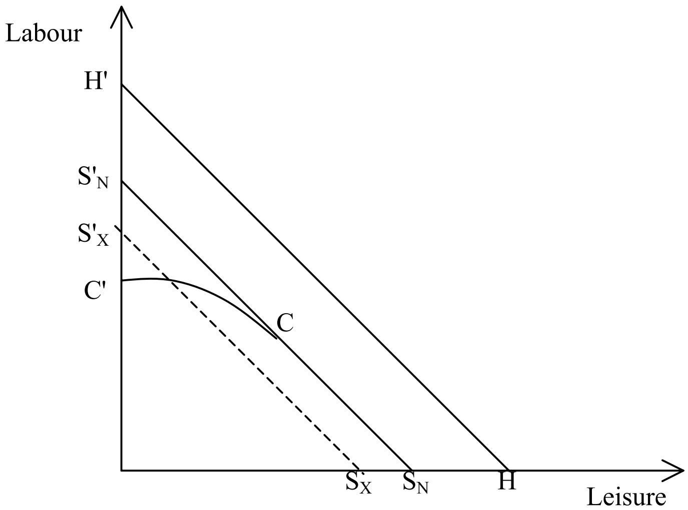
2.2 Efficient Time Opportunity Frontier

With the insights from the previous paragraph we now turn to the efficient time opportunity frontier, and explore how it differs when an individual is healthy the entire period and when he or she is ill part of the period.

Let us consider efficient time, which is the full productivity time-equivalent (i.e. time available multiplied by the level of productivity), and assume that the productivity-level under sickness is less than under healthy conditions. In a healthy period, the choice of time-use does not affect the productivity level, and time can therefore be allocated to either work or leisure with total efficient time staying constant. The line HH' in figure 1 shows this individual's opportunity set.

When an individual is sick part of the period, on the other hand, productivity will be lower during the sickness period and this will decrease the total efficient time available. Hence, the efficient time opportunity frontier during a period with sickness will lie to the left of HH'. If sickness time was considered as time lost, i.e. a time used merely to recuperate with recuperation not providing any form of utility as in Grossman and O'Donnell's models, then choice would only be available over the use of the remaining healthy time during the period. Since healthy time in our model is not used for health investments, in contrast to the Grossman tradition, time use would not matter for total efficient time and the budget line would be parallel to HH' (denoted by $S_X S'_X$ in figure1).

Figure 1: Efficient time opportunity frontiers for a healthy period, and in the case of exogenous and endogenous sickness periods.



However, in the model developed here sickness time is not assumed entirely lost. In particular, it is assumed that whether sickness time is spent recuperating at home or continuing working has an effect on the duration and severity of the illness, and that sickness time spent recuperating provides a certain amount of utility in terms of leisure. If the full recuperation time is spent recuperating, then we know that the post-recuperation time will be fully productive. Hence the loss in efficient time occurs only during the recuperation period in which the individual derives less than full leisure-utility, we will call this the *direct* effect. For all choices of leisure and labour where the individual chooses to fully recover, the efficient time opportunity set will therefore be an inwards-shifted version of the healthy one. This is illustrated by line segment $S_N C$ in figure 1.[8]

If part or all of recuperation time is spent working there will be an *indirect*, as well as a direct effect on the amount of total efficient time. The direct effect is again the reduced productivity in any activity during the full recuperation time. The indirect, or endogenous, effect comes from the reduction in post-recuperation period productivity due to not having recuperated fully. This effect will depend on the extent of actual recuperation, as well as on the severity of the sickness, and the efficiency of recuperation time in producing post-recuperation healthy time. Hence, the amount of total efficient time will depend on how much of the recuperation time the individual chooses to work.

With these insights we can now derive the efficient time opportunity frontier for a period that involves some sickness. For efficient leisure time at least as large as efficient recuperation time, the opportunity frontier under sickness is just a parallel inward shift of the healthy opportunity frontier. For efficient leisure time less than efficient recuperation time, the more of the recuperation time spent working the less the total efficient time i.e. the time opportunity frontier is convex to the origin. This is illustrated by line $C'CS_N$ in figure 1.

Assume that the consumer only cares about leisure and consumption. This implies that for any given amount of leisure, the consumer would want to get the maximum consumption possible and thus the maximum amount of efficient work $-\Phi(L)$. The amount of efficient time, and the allocation to efficient work and efficient leisure, will be a function of the choice of actual recuperation during the full recuperation time, x, and of the choice of leisure in the post-recuperation time, z.

[8] The line segment is not shifted as much inwards as was the case under the assumption of sickness-time being time lost, because some leisure-utility is now derived from recuperating.

Define the maximisation problem in the following way:

$$\Phi(L) \equiv MAX\ s\ (1-x)\ r + \sigma(x)\ (1-z)\ (1-r) \quad (3)$$

$$\text{s. t.} \quad s\ x\ r + \sigma(x)\ z(1-r) \geq L,\ 0 \leq x \leq 1,\ 0 \leq z \leq 1$$

The solution to this problem determines *x* and *z* conditional on the value of *L* and thus the amount of recuperation a consumer will choose *L*. We form the following Lagrangean:

$$Z = s(1-x)r + \sigma(x)(1-z)(1-r) + \lambda\left(sxr + \sigma(x)z(1-r) - L\right) + \mu(1-x) + v(1-z)$$

Hence, the maximisation problem gives the following first order conditions (FOCs):

$$\partial\Phi/\partial x: (\lambda-1)\ sr + \sigma'(x)((1-r)(1-z) + \lambda(1-r)\ z) \leq \mu,\ x \geq 0 \text{ and } x[\partial\Phi/\partial x]=0$$

$$x \leq 1,\ \mu \geq 0 \quad \text{and} \quad \mu[1-x]=0$$

$$\partial\Phi/\partial z: (\lambda - 1)\ \sigma(x)\ (1 - r) \leq v, \quad z \geq 0 \quad \text{and} \quad z[\partial\Phi/\partial z]=0$$

$$z \leq 1,\quad v \geq 0 \quad \text{and} \quad v\ [1-z]=0$$

$$\partial\Phi/\partial\lambda: s\ r\ x + \sigma(x)\ (1 - r)\ z \geq L, \quad \lambda \geq 0 \quad \text{and} \quad \lambda[\partial\Phi/\partial\lambda]=0$$

We start by analyzing the case where $z > 0$, i.e. where part of the post-recuperation period is spent in leisure, which from here onwards will be called ‘regime 1’. The second type of regime, ‘regime 2’, occurs when no leisure is taken during post-recuperation (i.e. $z = 0$).

Regime 1: Part of Post-Recuperation Time Spent in Leisure

Let us first look at the case where part of, or the entire, post-recuperation phase is spent in leisure, i.e. $1 \geq z > 0$. This implies that $v \geq 0$ and $[\partial\Phi/\partial z] = 0$. Plugging these values into the second first order condition (FOC) we see that $\lambda \geq 1 > 0$. Furthermore, from the first FOC we find that $\lambda \geq 1$ in turn implies that $\mu > 0$ which, due to the complementary slackness requirement, implies $x = 1$. By substituting these values for recuperation and post-recuperation leisure, *x* and *z*, into equations (1) and (2), this procedure gives us the following expression for leisure and labour:

$$L = s\ r + (1-r)\ z \qquad (s\ r < L \leq s\ r + (1-r)) \quad (4)$$
$$\Phi\ (L) = s\ r + (1 - r) - L = 0$$

From these equations we see that total efficient time, *T*, which is equal to efficient labour plus efficient leisure, will be given by:

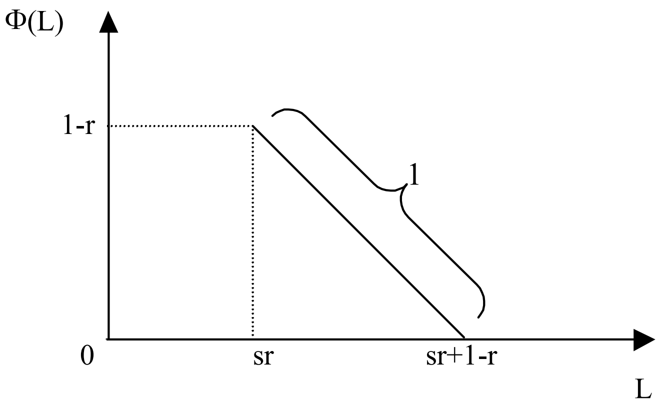
$$T = L + \Phi\ (L) = 1 - r\ (1-s) \quad (5)$$

Differentiating this expression with respect to efficient leisure yields the slope of the efficient time budget constraint: $\Phi'\ (L) = -1$. As expected, the efficient time budget constraint is negatively sloped in the efficient labour and efficient leisure space. A value of -1 means that for the type of individuals that choose to recuperate fully, each additional hour of efficient leisure can be substituted for an hour of efficient labour. This type of regime is illustrated by the line denoted 1 in figure 2 below.

The main insight from the previous analysis is that if any leisure is taken in the post-recuperation phase then recuperation is spent entirely in leisure (recuperating). This possibility arises if and only if $L > s\ r$. The intuition behind this result is clear; if an individual spends any time on leisure he should rather spend it during recuperation than post-recuperation time, because the former choice will at least improve the post-recuperation productivity whereas the latter will not.[9]

[9] We are here assuming a flexible work situation allowing the individual to take the full recuperation period out in leisure and to work long hours when healthy.

Figure 2: Efficient time opportunity frontier for those who take some or all of post-recuperation time out in leisure.



Regime 2: No Post-Recuperation Time Spent in Leisure

The second type of regime occurs when $0 \le L \le s\ r$, i.e. when no leisure is taken during post-recuperation ($z = 0$). This solution type is characterized by: $0 \le x \le 1$, which implies $\mu \ge 0$ and $[\partial \Phi / \partial x] \le 0$. Substituting these values into the FOCs we find that $\lambda < 1$, which implies $z = 0$. A marginal effect on labour of giving up one unit of leisure smaller than one implies that less efficient labour can be obtained by trading in efficient leisure. These worsened terms of trade are not due to characteristics specific to the individual, but to the characteristics of the illness.

In order to really understand what determines the shape of the budget constraint when an amount of efficient leisure is chosen which is less than the point denoted $s\ r$ in figure 1 (i.e. $0 \le L \le sr$), the maximization problem will now be simplified. Knowing that when part of recuperation time is spent working there will be no post-recuperation time leisure, let us define our problem as one of maximizing consumption/work given that $z=0$:

$$\hat{\Phi} \equiv \underset{0 \le x \le 1}{Max}\ s\ r(1 - x) + \sigma(x)(1 - r) \qquad (6)$$

The maximization problem gives the following first order condition:

$$\partial \hat{\Phi} / \partial x\text{: } - s\ r + \sigma'(x)\ (1 - r) \le \mu, \quad x \ge 0 \quad \text{and} \quad x\ [\partial \hat{\Phi} / \partial x] = 0$$

$$x \le 1, \quad \mu \ge 0 \quad \text{and} \quad \mu\ [1 - x] = 0$$

Based on this first order condition we can now distinguish in more detail between 2 different cases which have an impact on the efficient time frontier, each determined by the characteristics of the illness.[10]

Case 1: In this case we investigate the conditions under which exact full recuperation, i.e. $x=1$, yields the maximum amount of obtainable efficient labour. From the Kuhn-Tucker conditions above we find that $x=1$ implies $\mu \ge 0$ and $[\partial \hat{\Phi} / \partial x] = 0$. This then yields the following condition for case 1:

$$\sigma'(1) \ge \frac{sr}{1 - r} \qquad (7)$$

[10] Please refer to appendix A for a third case, which is a hybrid of the two pure cases.

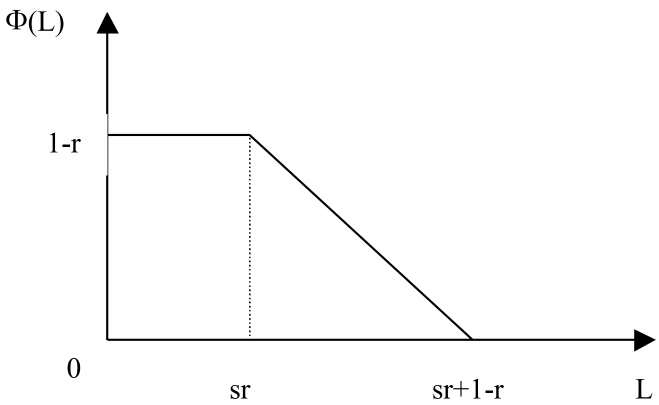
Note that $\sigma'(x)$ is the marginal productivity of an increase in actual time off during recuperation time, measured in terms of the extra productivity in post-recuperation time. Case 1 implies that it always pays to take time off during the recuperation period, even at the point where all recuperation time is spent at home.

In this case the solution to the maximization problem in (6) occurs when $x=1$, and $\hat{\Phi}=1-r$. Here $\Phi(L)=\hat{\Phi}=1-r$ $(\forall L,\ 0 \le L \le s\,r)$, which means that the entire recuperation period is spent recuperating and the entire post-recuperation period is spent working (see figure 3). Notice that, by concavity, $\sigma'(1)<\sigma(1)-\sigma(0)=1-s$. Using this inequality, together with expression (7), we find that a necessary condition for this case to arise is that $s < 1-r$. We can conclude that this case will be more likely to occur when the marginal productivity during recuperation is relatively low.

Figure 3 depicts the efficient time constraint for case 1. Maximum leisure is obtained when the person stays at home the entire recuperation period (equivalent of $s\,r$ units of efficient leisure) and thus obtains full efficient leisure in the post-recuperation period, $1-r$ (because $\sigma(x) = 1$): $L = s\,r + 1-r$. Since any unit of work during the recuperation period will be at least offset by the ensuing decrease in post-recuperation productivity at work, the maximum amount of work that can be obtained is the equivalent to the entire post-recuperation period.

Hence, at any point on the leisure axis below sr, efficient leisure derived from staying at home during recuperation, the maximum obtainable amount of efficient work will be $1-r$.

Figure 3: Efficient time opportunity frontier in the case where exact full recuperation yields the maximum amount of efficient labour (case 1).



Case 2: The second case considered is when the maximum efficient labour is obtained by not taking any time off in order to recuperate, i.e. $x = 0$. From the Kuhn-Tucker conditions above we find that $x = 0$ implies $\mu = 0$ and $[\partial \hat{\Phi}/\partial x] \le 0$. This then gives us the following condition for case 3 to arise:

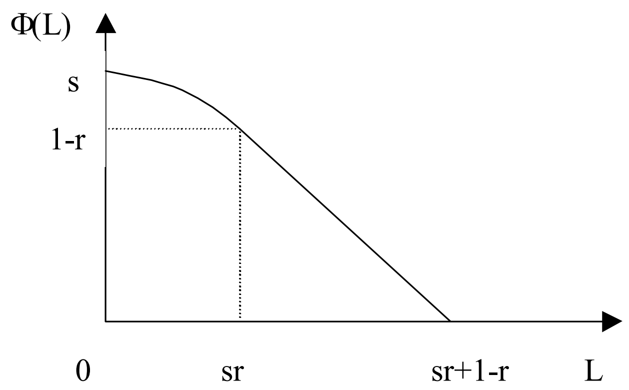
$$\sigma'(0) \le \frac{sr}{1-r} \tag{8}$$

This case implies that the marginal productivity in the post-recuperation phase due to an increase in actual recuperation during recuperation time multiplied by the length of the post- recuperation period is smaller than the productivity during the recuperation phase multiplied by its length, even at the point where no recuperation time is spent recuperating. Then the maximisation of (6) occurs when $x = 0$, and so $\hat{\Phi} = s$. Notice that by concavity $\sigma'(0) > \sigma(1) - \sigma(0) = 1 - s$.

When combining this finding with the initial condition (i.e. $s\, r / (1 - r) \geq \sigma'(0)$), we see that a necessary condition for this case to arise is that $s > 1 - r$ (see figure 4). This case will therefore be more likely when the marginal productivity during recuperation is rather high. Maximum efficient labour for all L, $0 \leq L \leq s\, r$, is here given by: $\Phi(L) = s\, r - L + (1 - r)\, \sigma(L / s\, r)$.

Figure 3 depicts the time constraint under the case 2. Any amount of efficient leisure during the recuperation time can now be traded against additional work time, with a positive effect on overall efficient work time. If the person works the entire recuperation period, he or she will get an amount of efficient work time equal to $s > 1 - r$.

Figure 4: Efficient time opportunity frontier in the case where working during the entire recuperation time yields the maximum amount of efficient labour (case2).



The efficient time frontier has hence been divided into two cases distinguished by their illness characteristics. Illness case 1 arises when productivity during the recuperation period is relatively low - certainly the efficient labour obtained from working the entire period (*s*) is less than what would be obtained by fully recuperating and working the entire post-recuperation period at full productivity (*1-r*). In this case everyone will spend the full time recuperating. Illness case 2 predicts that the recuperation time can be traded against additional work time with a positive effect on overall efficient work time, and therefore that some consumers will choose to not fully recuperate. Which of the two cases is the correct description of reality will depend on the type of illness and type of work considered. Each of these illness cases consists of two main types of individuals; those that choose to have relatively much leisure time (and to at least recuperate fully) and those that choose to have relatively little leisure time (and to recuperate less than or exactly fully).

2.3 Utility Maximisation

When deriving the efficient time budget constraint we found that there is a tendency for individuals who choose higher amounts of leisure to fully recuperate. The question that now will be explored is therefore what type of individuals tend to spend more time in leisure? The only feature that distinguishes the individuals in this model is the wage rate, so the related question is whether wage has an effect on the amount of efficient leisure chosen. In order to answer this question, we now turn to maximizing utility. Utility, which is assumed to be a function of consumption and efficient leisure, will be maximized subject to the budget constraint, using the insights about maximum obtainable efficient labour time derived in the previous section. The main aim in this part of the analysis is twofold: first, to find the marginal effect of wage upon leisure, and second, to analyze whether a critical wage exists above which individuals choose to recuperate fully.

Assuming that people have a minimum consumption requirement on wages, consumption is split into two types; the necessary minimum requirement, *m*, and the consumption over which we have a choice, *c*. The individual further has unearned income, $\bar{y}$, and wage earnings, $w\Phi(L)$, where $\Phi(L)$ is the maximum efficient labour obtainable for a certain level of efficient leisure, as derived in section 2.2. Define $a = m - \bar{y}$, where a is the *net* minimum consumption constraint. We then have the following expression for consumption:

$$c \le w\ \Phi(L) - a \tag{9}$$

For the sake of simplifying the exposition we will use the CES utility function, however, the main conclusions will not significantly change for the more general case of an additively separable utility function.[11] Let us take the following CES utility function:[12]

$$U(c,L) = \begin{cases} \frac{\alpha}{1-\lambda}[C]^{1-\lambda} + \frac{1-\alpha}{1-\lambda}[L]^{1-\lambda}, \lambda \ge 0, \lambda \ne 1 \\ \alpha \log(c) + (1-\alpha)\log(L), \lambda = 1 \end{cases} \tag{10}$$

where $0 < \alpha < 1$, and $\sigma = 1/\lambda$ is the elasticity of substitution. The Cobb Douglas case is where $\sigma = \lambda = 1$. Maximising this utility function with respect to the budget constraint can therefore be expressed as:

$$\underset{0 \le L \le 1}{Max} U = \frac{\alpha}{1-\lambda}[w\Phi(L) - a]^{1-\lambda} + \frac{1-\alpha}{1-\lambda}[L]^{1-\lambda} \tag{11}$$

The first order condition is:

$$\alpha w \Phi'(L)(w\Phi(L) - a)^{-\lambda} + (1-\alpha)L^{-\lambda} = 0 \tag{12}$$

Then, following the envelope theorem, the marginal effect of wage upon leisure, i.e. the sign of $dL(w)/dw$, will be equal to the sign of the following expression:[13]

$$-\left\{1 - \left(\frac{w\Phi(L)}{w\Phi(L) - a}\right)\lambda\right\} \tag{13}$$

Define $\mu = \frac{w\Phi(L)}{(w\Phi(L) - a)}$, where μ is the ratio of overall consumption to choice consumption, the latter being given by total income net of the net minimum consumption requirement. Whether the expression $\lambda\mu - 1$ overall has a positive or negative sign thus depends on whether $\lambda\ \mu = \mu/\sigma > 1$ or not. The ratio of overall consumption to choice consumption is always larger or equal to one, $\mu \ge 1$. When the wage rate goes to infinity

[11] Refer to appendix B for the mathematical derivation.

[12] The textbook CES-function is often defined as: $U = \left[a_1 x_1^{\ \rho} + a_2 x_2^{\ \rho}\right]^{\frac{1}{\rho}}$. Since preferences are invariant with respect to monotonic transforms of utility, we can just as well use equation (10). Note: $\rho = 1-\lambda$. The term $1-\lambda$ in the denominator is needed to ensure that the utility function is strictly increasing in c and L (i.e. that the marginal utilities of consumption and leisure are positive even in the case where $\lambda > 1$).

[13] For the calculation see the appendix B.3.

μ goes to one (i.e. $\mu \to 1$ if $w \to \infty$), and when the wage rate goes to the existence wage (w $\to w_{min} = a/\Phi(L)$) μ goes to infinity (i.e. $\mu \to \infty$ if $w \to w_{min}$).

The higher λ, and thus the lower the elasticity of substitution between consumption and leisure, the more rapid the proportionate decline in the marginal utility of consumption in response to increases in c. As long as the elasticity of substitution is smaller than one, leisure will increase in wages since the ratio of total consumption to choice consumption was found to be larger than one. Furthermore, the closer the wage rate comes to the existence wage, the higher will be the elasticity of substitution that still allows leisure to be increasing in wages.

Based on econometric work on micro data from Norway by Aaberge et al. (1995) the elasticity of substitution between consumption and leisure was found to be 0.25. Pencavel (1986) reviewed a number of empirical studies with elasticity estimates for men in the United States and Britain. The substitution effect was found to fall in the range of 0.1 to 0.2 in Britain, and to be about 0.1 for the United States. Blundell and MaCurdy (1998) similarly summarized the results from a number of empirical studies and found that the elasticity from the majority of the studies was positive and smaller than 1.

We know from the analysis in section 2.2 that the wage group that will choose to at least fully recuperate fulfils the condition of $L \geq s\ r$. In order to analyse the conditions that determine the existence of a critical wage above which people choose to fully recuperate, we hence need an expression for the amount of leisure chosen by those who at least recuperate fully. Using equation (5) to substitute for $\phi(L)$ and $\phi'(L)$ we can derive the expression for leisure from equation (12):

$$L = \frac{wT - a}{\gamma w^{\sigma} + w} \tag{14}$$

where $\gamma = \left(\frac{\alpha}{1-\alpha}\right)^{\sigma}$ and $a > 0$ is the net minimum consumption. Hence, we focus on the case where there is a critical wage, $\overline{w}$, for which $L(\overline{w}) = \overline{L} = sr$, and $L(w) > \overline{L}$ for all wages above the critical wage ($\forall w > \overline{w}$). $\overline{L}$, $0 < \overline{L} < T$, is the critical value of leisure above which people choose to fully recuperate. By differentiating expression (14) with respect to the wage rate, we find that leisure demand is strictly increasing in wage as long as we assume $\sigma \leq 1$.

From equation (14) we observe that if the income obtained by working the entire period exactly equals the net minimum consumption requirement, then no leisure is taken (i.e. $L=0$ when $w\ T = a$), whereas when wages tend towards infinity leisure time tends towards total time (i.e. $L \to T$ as $w \to \infty$). Hence there always exists a unique $\overline{w}$ for which $L(\overline{w}) = \overline{L}$.[14, 15]

We have seen in this section that, given a CES utility function and an empirically supported elasticity of substitution between leisure and consumption, and assuming that the minimum consumption constraint on wage income net of unearned income is non-negative, there will be a tendency for an increase in the wage to increase the demand for leisure. In this case the analysis suggests that people with low wages will be working

[14] In the Cobb-Douglas case, where $\sigma = 1$, we see that once again no leisure is taken when earning the existence wage (i.e. $L = 0$ when $w\ T = a$), whereas when wages tend towards infinity leisure time now tends towards an amount less than total time (i.e. $L \to T/(1+\gamma)$ as $w \to \infty$). In order to ensure that there is a critical wage $\overline{w}$ for which $L(\overline{w}) = \overline{L}$, it is in this case necessary to assume that $\overline{L} \leq T/(1+\gamma)$.

[15] For the derivation of the critical wage, and the wage ranges in both illness cases refer to appendix C.

during sickness time (as long as it produces additional efficient labour), which in turn will imply a lower productivity in their post-recuperation phase.

3. Measuring the Adverse Effects from Illness

The implication of this finding for traditional illness measurements, such as work absence, as well as the income and welfare implications are discussed in the next section, and contrasted with a more traditional scenario where illness is assumed time lost from any activity. The latter model will be known as the exogenous recuperation model (EXOR).[16] In order to simplify the mathematical exposition, we will look at the specific CES-utility case where $\lambda=1$, i.e. the Cobb-Douglas (C-D) case from here onwards.

3.1 Work-absence

One commonly used statistical measurement of illness has been absence from work due to illness, also known as work loss days (e.g. Zuidema and Nentjes (1996), Ostro (1983), Hansen and Selte (2000)). Work-absence will capture the sickness of those who choose to take time off to recuperate, as well as of those who are forced to recuperate because of the severity of the illness. In this paper work-absence (WA) is defined as absence from work during which the individual is sick, and absence is measured as the amount of labour supplied during the illness subtracted from the amount the person would have been working during that period had he or she been healthy.[17] The fact that the individual may be compensating by working more when recuperated is not accounted for in this measure.

In the ENDOR model, only higher-wage individuals *choose* not to work while being sick, whereas sickness time is *time lost* from any activity for all wage groups in the EXOR model. When using work-absence as a measure of illness, this distinction is lost. We can obtain the analytical expression for work-absence due to illness, both in the EXOR case and the regime 1 endogenous case, by multiplying the fraction of any time period that the individual would have worked when healthy, with the amount of recuperation:

$$WA = r\left(\alpha\left(1-\frac{a}{w}\right)+\frac{a}{w}\right) \qquad \text{with} \qquad \frac{\partial WA}{\partial w}<0 \tag{16}$$

The α denotes the weight given to choice consumption in the utility function, and a is the net minimum consumption requirement. The amount of absence from work will decrease with increasing wages due to the fact that the amount of leisure time chosen increases with wages, and corollary with this finding the time worked decreases with wages and therefore the working time lost due to illness.

In the severe illness case (case 1), the low-wage individuals will be absent from work during their entire illness, and the amount of absence measured will depend on how much they would have worked during that period had they been healthy. Absence from work will again be given by the expression (16). Because of their low wages individuals would have been working more had they been healthy, and measured work-absence is therefore larger.

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[16] Note that we will assume the time loss to be equal across wage groups. In the health capital literature, the time lost to illness is endogenous and a function of investment in health capital, as well as the rate of depreciation. Several of these studies have pointed out that the time lost is likely to vary with individual's wage rates (e.g. Grossman (2000), Cropper (1977)).

[17] The period when they are ill would have been like any other time period under healthy conditions, hence it would have been used partly for leisure, partly for choice consumption, and partly to fulfil the net minimum consumption requirement during that sub-period.

In the less severe illness case (case 2), a trade-off exists for low-wage individuals between efficient leisure and efficient work during illness, and this wage-group hence works while being sick. Because the low-wage individuals in this less severe illness case will work while being ill, estimates of the adverse health effect based on work-absence will under-represent the adverse health effect from illness. Work-absence will in case 2 be given by the following expression:

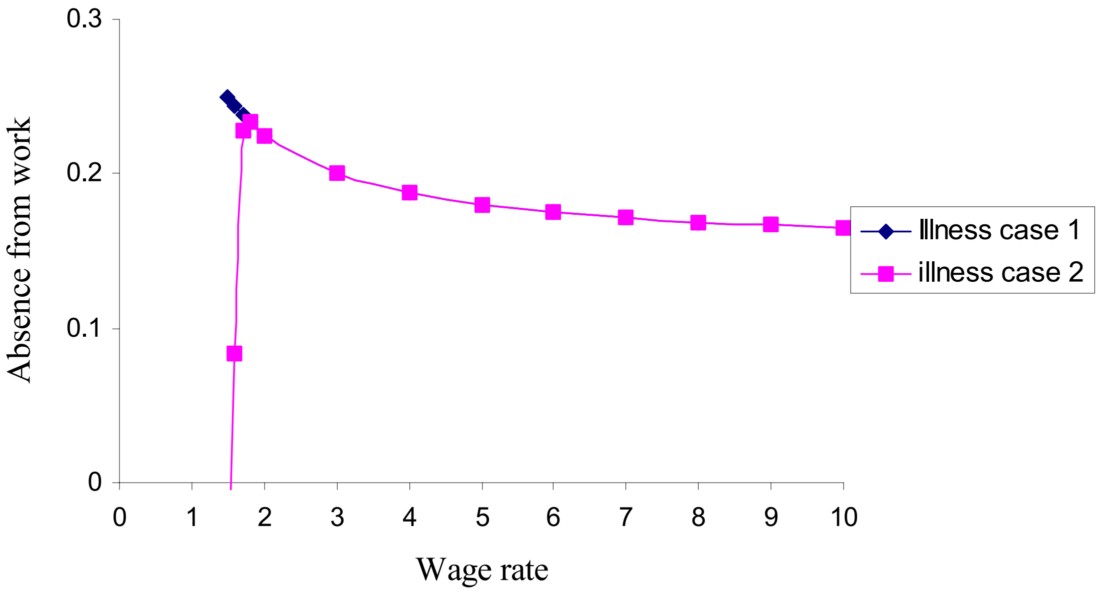
$$WA_{2,2} = r\left(\alpha\left(1-\frac{a}{w}\right)+\frac{a}{w}-(1-x)\right) \tag{17}$$

where x is the amount of actual recuperation chosen during the full recuperation period (i.e. the part of the recuperation period not spent working). The subscript "2, 2" denotes that we are looking at the low-wage individuals (regime 2) in illness case 2. By setting expression (17) equal to zero we can derive the wage rate at which no WA will be registered:

$$w = \frac{a(1-\alpha)}{(1-\alpha-x)}$$

An individual earning a wage rate lower than this will actually work more during the recuperation period than he/she would have done had he/she not been ill during that period (shown in figure 5). Therefore, work-absence would most severely misrepresent the adverse health effects for the low-wage individuals in illness case 2.

Figure 5: Work-absence at various wage rates for illness cases 1 and 2.



Note: The figure above is based on the following assumptions: a duration of the full recuperation, r, of 0.3, a weight on consumption in the utility function, α, of 0.5, and a net minimum consumption requirement, a, of 1. In addition we have assumed the productivity during illness, s, in the case 1 to be 0.5 and in the case 2 to be 0.8.

3.2 Income Loss

The loss experienced due to illness may be better captured by calculating income loss because this latter picks up the removal of the opportunity to earn income in illness case 1, and the constraint on this opportunity in illness case 2. Income losses are the subject of this section.

Another commonly used measurement of adverse health effects looks at the change in discounted life-time earnings and personal and societal expenditure on medicines and medical treatment. The latter are not considered in the present model, and since the model is essentially a one period model this Cost-Of-Illness (COI) approach therefore boils down to measuring the income loss caused by illness.

Income loss as a measurement of the effect of illness differs from work-absence in one important aspect. Since workers are paid according to productivity, income loss due to illness records the adverse effect on individuals who stay at home and also those who go to work. Nevertheless, income loss is not only interesting as an illness measurement. Policy-makers are increasingly paying attention to the effects of different policy-measures on income-distribution, and the income distributional effects of illness, and of any policies that affect illness or labour supply behaviour, are therefore of interest in their own right.

For those individuals who have a wage-rate that allows them to recuperate fully (denoted with subscript 1), the following expression for absolute income loss can be derived using equations (5) and (14):[18]

$$AIL_1 = \alpha\, w\, r\, (1-s) \qquad \text{with} \qquad \partial AIL / \partial w = r(1-s)\alpha > 0 \qquad (18)$$

Absolute income loss increases with higher wages for those individuals who have a wage above the critical wage due to the fact that the efficient work time loss is the same for all within this group because the weight on choice labour is not affected by the wage rate, and therefore the difference in income loss is determined by the wage rate with which each hour is paid. Relative income loss (which is here modelled as AIL divided by income in the healthy scenario), which is identical to the relative loss in efficient working hours, is higher for higher wages due to the fact that the amount of working-hours lost relative to the total amount of working hours is high for high wages, whereas the relative income loss is lower for lower wages since the hours lost is a smaller amount compared to the overall amount of hours worked.

The only difference between the income losses derived for these higher-wage individuals in the endogenous recuperation model and the income losses of all wage-groups in the EXOR model is that the parameter of productivity during illness, s, is not relevant in the latter and can be put equal to zero. Hence, both the absolute and relative income losses are larger by a factor of $1/(1-s)$.

For low-wage individuals in the severe illness case (case 1), the expression for absolute income loss is derived from equation (14) and the case 1 (see section 2.2):[19]

$$AIL_{2,1} = wr - (1-\alpha)(w-a) \qquad \text{with} \qquad \frac{\partial AIL}{\partial w} = \alpha - (1-r) = ? \qquad (19)$$

AIL is decreasing (increasing) in wages if the weight given to labour out of the available "choice" time when healthy, α, is smaller (larger) than the amount of efficient labour supplied when ill, here; $1-r$. In other words, if the amount of additional income made under healthy conditions with an increase in the wage rate is smaller than the additional income that would be made under illness, then the absolute income loss due to illness will decrease with increasing wages. The relative income loss would in this case be larger the lower the wage *within* this group due to the fact that income after illness is compared to lower initial income in the case of the poorer, and the minimum consumption requirement on wage earnings therefore matters more.

[18] See appendix D for the calculation steps.
[19] Ibid.

For low-wage individuals in the less severe illness case (case 2), the expression for absolute income loss would be the following:[20, 21]

$$AIL_{2,2} = w\alpha - \alpha a + a - w\left(sr - L + (1-r)\sigma\left(\frac{L}{sr}\right)\right) \qquad (20)$$

By substituting for wage (w) and leisure (L) in the special cases of *x=0* and *x=1*, we find that whether AIL and RIL decrease or increase when moving from the lower to the higher wage depends on the characteristics of the illness, i.e. the curvature of the marginal productivity in the post-recuperation period of additional recuperation, β, its duration, r, its severity, *1-s*, as well as on the taste of the individual (i.e. α).

In measuring the income loss due to illness, we have here seen that the traditional way of modelling illness as time lost will seriously misrepresent the adverse effect of illness if the duration of illness is in fact a choice variable (see figures 6 and 7). The EXOR model predicts that both absolute and relative income losses increase with increasing wages. However, we have in this section seen that income losses, both relative and absolute, incurred from illness may be higher for low-wage groups than for higher wage groups, illnesses may thereby have income-distributional implications.

Figure 6: Income losses (absolute and relative) at various wage rates for illness case 1 and the EXOR model.

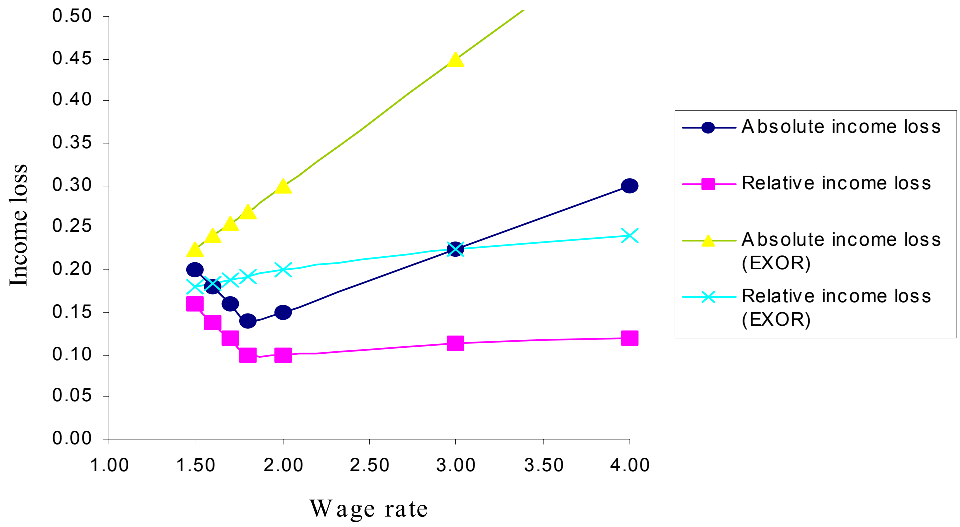
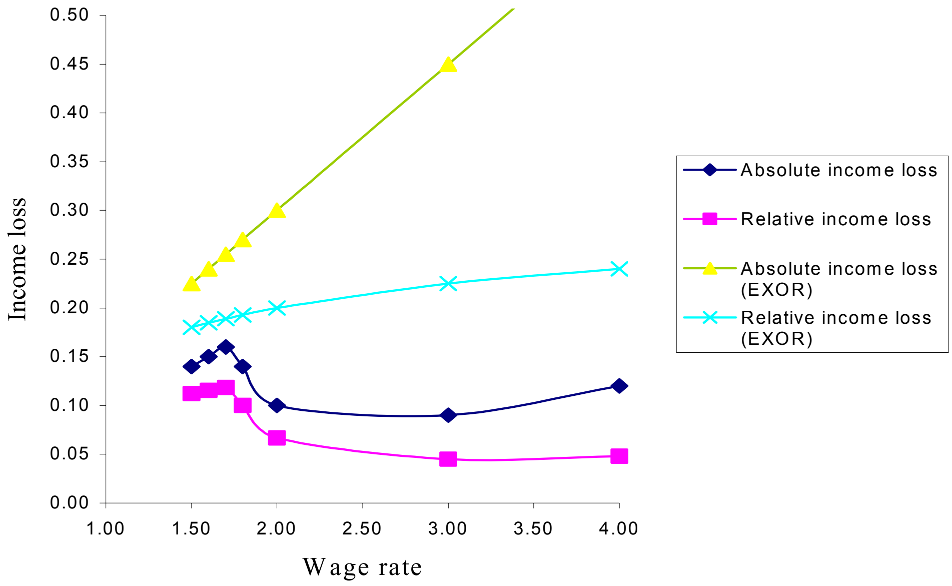


Figure 7: Income losses (absolute and relative) at various wage rates for illness case 2 and the EXOR model.

[20] Ibid.

[21] For those individuals within the low-wage group that choose to locate at the kink of the efficient time budget constraint in illness case 2 (i.e. those who have a wage between the lower kink-wage and the critical wage – see appendix C), the expression for AIL will be the same as that for case 1 given in expression (19).



Note: The figures above are based on the following assumptions: a duration of the full recuperation, *r*, of 0.3, a weight on consumption in the utility function, α, of 0.5, and a net minimum consumption requirement, *a*, of 1. In addition we have assumed the productivity during illness, *s*, in the case 1 to be 0.5 and in the case 2 to be 0.8.

3.3 Welfare Loss

In this section we will show that individuals with the lowest wages may loose more welfare absolutely than individuals with higher wages up to a certain wage level due to the removal or restriction on the opportunity to earn additional income during illness. In the case of the EXOR model, on the other hand, the absolute welfare losses increase in wages throughout. Figures 8 and 9 at the end of this section illustrate these very different model predictions. Relative welfare loss due to illness will be lower the higher the wage rate of the individual considered in both models.

If we want to have a proper measurement of the full adverse effects of illness we will have to take into consideration both consumption and leisure. Preston and Walker (1992) review a number of welfare measures that can be used in the context of non-linear budget constraints.[22] We have chosen here the full income expenditure function at a reference wage, since it is the classic money metric of utility that can be computed and interpreted easily, and compared directly with net income.

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[22] Preston and Walker (1992) noted that most of the literature that has taken leisure into account when measuring welfare has simply treated it as an additional commodity, overlooking the fact that what distinguishes the analysis of labour supply from that of commodity demands is the nonlinearity associated with the budget constraint. Nonlinear budget constraints can arise under different circumstances such as nonlinear income taxes (Hausman (1985), Preston and Walker (1992)), non-linear household production functions (Pollack and Wachter (1975)), non-linear wage-hours locus (Deardorff and Stafford (1976)), and, finally, endogenous recuperation (Gaarder (2002)). In addition to the classic full income expenditure function, which measures the value of a consumer's consumption plus her leisure (which are equal to her endowment of consumption and time), Preston and Walker review a number of welfare measures that can be used in the context of non-linear budget constraints: (i) The unearned income expenditure function which gives the income at zero hours required to attain some utility level at a reference wage (and therefore differs from the full income measure by the value of the time endowment); (ii) the consumption level (of the composite commodity) required to attain some utility level at a given reference wage; (iii) the consumption level required to attain some utility level at a reference level of hours of work; (iv) the wage required to attain some utility level at a reference level of unearned income; and (v) the consumption level required to attain some utility level at a reference level of unearned income.

The following analysis draws on King (1983) and Aaberge and Colombino (1998) by deriving measures of welfare from equivalent incomes defined in terms of a reference household (here individual) and the prices (here reference wage) this household faces. The introduction of a common reference wage as a basis for comparing welfare across individuals is motivated by the fact that real/efficient wage, the price of leisure, will vary across individuals as well as between states (here healthy state and state of illness).[23]

Although we have distinguished between wage and efficient time in the preceding sections, it is important to realise that *efficient* wage is the wage received per nominal hour (in contrast to efficient hour) and may therefore vary between the healthy and sick state, as well as between non-poor and poor individuals.

The indirect utility function of full income and wage is given by maximising the direct utility, which is a function of consumption and leisure, subject to the budget constraint:

$$v(w,F) \equiv_{MAX} u(C,L) \ s.t. \ C + w\,L \leq F$$

where v is the indirect utility function, w is wage, F is full income, u is the direct utility function, C consumption, and L leisure.

The approach for determining the change in welfare of one particular individual due to illness is to employ monetary measures defined in terms of money values of indirect utilities. For given consumer prices and a wage the money metric utility, F_i^k, is defined implicitly by

$$V_i^k\left(F_i^k, w_i\right) = \underset{L}{MAX}\, u\left[w_i \Phi^h(L) - a_i, L\right], \quad k = h, s \qquad \text{and} \qquad i = r, p$$

where V_i is the indirect utility function of individual i and F_i^k is the full income of individual i in the state of health k. The wage and the minimum consumption requirement on wage earnings will vary over individuals, i. We will in addition distinguish between two states of health, k. The state of health denoted h implies that the individual is healthy the entire period, and the one denoted s that part of the period is spent being sick. Thus, the full income F_i^k affords individual i the same level of indirect utility under the wage w_i as the maximum amount of obtainable direct utility attained with wage w_i with full income F_i^k.

In order to make these money measures of utility comparable across individuals King (1983) suggested to base the comparison on equivalent incomes defined in terms of a reference household (here; individual) facing a reference price. Since the only factor separating different individuals in this model is the wage rate, this method implies in practice using the same reference wage for all individuals. Equivalent income, F_i^k, for individual i is then defined as that level of full income that yields the same level of utility at the reference wage, $\overline{w}$, as the same individual i attains under the wage w_i. F_i^k is given implicitly by the following expression:

$$v\left[\overline{w}, F_i^k\right] = V_i^k = \underset{L}{Max}\, u\left[w_i \Phi^k(L) - a_i, L\right], k = h, s \ \text{ and } \ i = r, p$$

[23] Both King (1983) and Aaberge et al. (1998) mention the possibility that the outcome of the comparison of welfare changes may depend on the choice of reference price, and it will be important to examine the sensitivity of the results with regards to the choice of reference wage. It is however worth noting that Aaberge et al. carry out such a sensitivity analysis and find that the main conclusions are not affected by the choice of reference state.

From this expression we see that the difference between F_i^h and F_i^s can be used as a measure of the welfare loss from illness to individual *i*, and as the money values are defined in terms of a fixed reference wage, this choice of measure allows for welfare change comparisons across individuals. We denote the measure *absolute comparable welfare loss* (ACWL):

$$ACWL_i = F_i^h - F_i^s \qquad (21)$$

The *relative comparable welfare loss* (RCWL) is given by ACWL relative to the equivalent income in the healthy state, F_i^h. The budget constraint given in expression (9) can be given by the following expression:

$$C + L\,w \leq F \quad \text{and} \quad F = T\,w\text{-}a \qquad (22)$$

where full income, *F*, is equal to total time, *T*, multiplied by the wage rate, *w*, minus *a*, the minimum consumption requirement on wage-earnings.

For those individuals who have a wage-rate that allows them to recuperate fully (denoted with subscript 1), the following expressions for absolute and relative welfare loss can be derived using equations (14) and (21), and (22):[24]

$$ACWL_1 = \left(\frac{\overline{w}}{w}\right)^{1-\alpha} wr(1-s) \qquad \text{with} \quad \frac{\partial ACWL}{\partial w} > 0 \qquad (23)$$

A higher wage leads to a higher absolute welfare loss. This is intuitively quite clear because those individuals with higher wages loose a higher amount of consumption (although the same amount of efficient labour hours), whereas the amount of leisure time lost is the same and this leads to a higher absolute welfare loss. Relative welfare loss, on the other hand, is decreasing in wage due to the fact that welfare after illness is compared to lower initial welfare in the case of the poorer, and the minimum consumption requirement on wage earnings therefore matters more.

The only difference between the welfare losses derived for these higher-wage individuals in the endogenous recuperation model and the income losses of all wage-groups in the EXOR model is that the parameter of productivity during illness, *s*, is not relevant and can be put equal to zero. Hence, both the absolute and relative income losses are larger by a factor of *1/(1-s)*, just as we found in the efficient time loss and the income loss calculations.

For low-wage individuals in the severe illness case (case 1), i.e. those who locate their choice of leisure time exactly at the kink, where $L = s\,r$ and $H = 1 - r$, the absolute and relative comparable welfare losses can be expressed as follows:[25, 26]

$$ACWL = \overline{w}^{1-\alpha} w^{\alpha} \left(1 - \frac{a}{w} - \frac{\left(1 - r - \frac{a}{w}\right)^{\alpha} (sr)^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \right) \qquad \text{with} \quad \frac{\partial ACWL}{\partial w} = ? \qquad (24)$$

Absolute welfare losses decrease with increasing wages when the additional wage leads to more additional welfare in the sick case than in the healthy case. With C-D preferences this is the case if the optimal amount of consumption in the healthy scenario is smaller

[24] See appendix E for the calculation steps.
[25] Ibid.
[26] See appendix E for differentiation w.r.t. wage.

than the amount of consumption chosen under illness case 1, multiplied by the marginal rate of substitution between efficient leisure and efficient labour in the sickness scenario to the power of ($1-\alpha$), i.e. the weight given to leisure in the utility function, i.e.:

$$\alpha\left(1-\frac{a}{w}\right)+\frac{a}{w}<\left(\frac{\alpha wsr}{(1-\alpha)(w(1-r)-a)}\right)^{1-\alpha}(1-r)$$

If we think of efficient labour here in terms of leisure equivalents, the expression above means that when the marginal rate of substitution is infinity in the sickness case (i.e. the individual would give up any amount of efficient leisure to obtain an additional unit of efficient labour) the increase in consumption due to a marginal increase in wage would be worth infinitely much in terms of leisure equivalents, and certainly more than an increase in consumption due to a marginal wage increase in the healthy case (which can be traded one for one with leisure). On the other hand, when the marginal rate of substitution is 1, efficient labour both in the sickness case and the healthy case can be substituted one for one with efficient labour, and in this case an increase in wage will obviously have a larger impact in the healthy scenario since the individual provides more efficient labour then. The condition above is thus fulfilled when the slope of the budget constraint is 0, but not when it is equal to –1 (at the critical wage). Some marginal rate of substitution between 0 and 1 will make the two sides of the inequality equal, i.e. at that rate of substitution the wage rate will not affect ACWL.

Relative welfare loss is decreasing with increasing wages when consumption in the healthy scenario exceeds consumption in the sickness scenario, i.e.: $\alpha(w-a)+a>w(1-r)$. This condition is necessarily fulfilled since consumption is assumed to be a normal good. Relative welfare loss is decreasing in wage due to the fact that reducing the net consumption constraint by increasing wages has a larger relative effect (positive) on welfare in the illness scenario than in the healthy scenario.

For the individuals who choose a bundle located on the curved part of the budget constraint (i.e. illness case 2), the absolute and relative welfare losses are given by the following expressions:[27, 28]

$$ACWL=\left(\frac{\overline{w}}{w}\right)^{1-\alpha}\left((w-a)-\frac{(wT^S-a)(-\Phi')^{\alpha}}{(1-\alpha-\alpha\Phi')}\right) \qquad (25)$$

The expressions for ACWL and RCWL are functions of the slope of the efficient time budget constraint, which is a function of actual recuperation time and thus of the wage rate. We can therefore only derive exact expressions for the absolute and relative welfare losses for the individuals with wages within the boundaries of this wage group numerically.

By substituting for wage (w) and leisure (L) in the special cases of $x=0$ and $x=1$, we find that whether AIL and RIL decrease or increase when moving from the lower to the higher wage depends on the characteristics of the illness, i.e. its duration, r, its severity, $1-s$, and the marginal productivity in the post-recuperation period of additional recuperation, $\sigma'(x)$, as well as the taste of the individual (i.e. the exponentials in the utility function, α). RCWL, on the other hand, decreases in wage.

[27] Ibid.

[28] For those individuals within the low-wage group that choose to locate at the kink of the efficient time budget constraint in illness case 2 (i.e. those who have a wage between the lower kink-wage and the critical wage – see appendix C), the expression for AIL will be the same as that for case 1 given in expression (24).

Figure 8: Welfare losses (absolute and relative) at various wage rates for illness case 1 and the EXOR model.

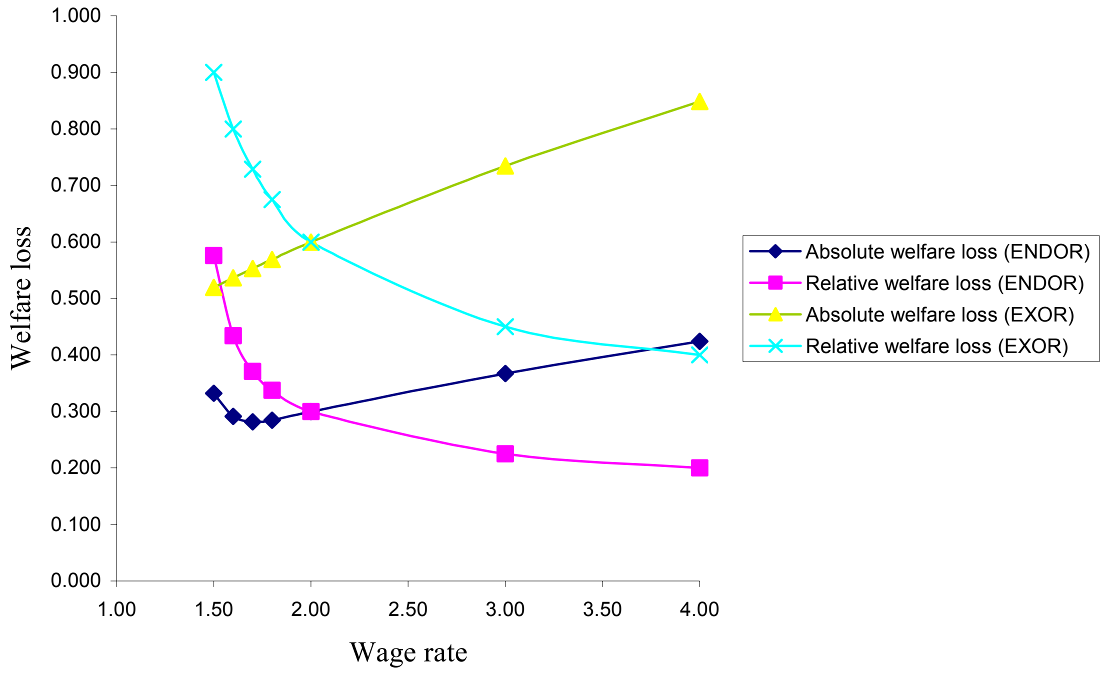
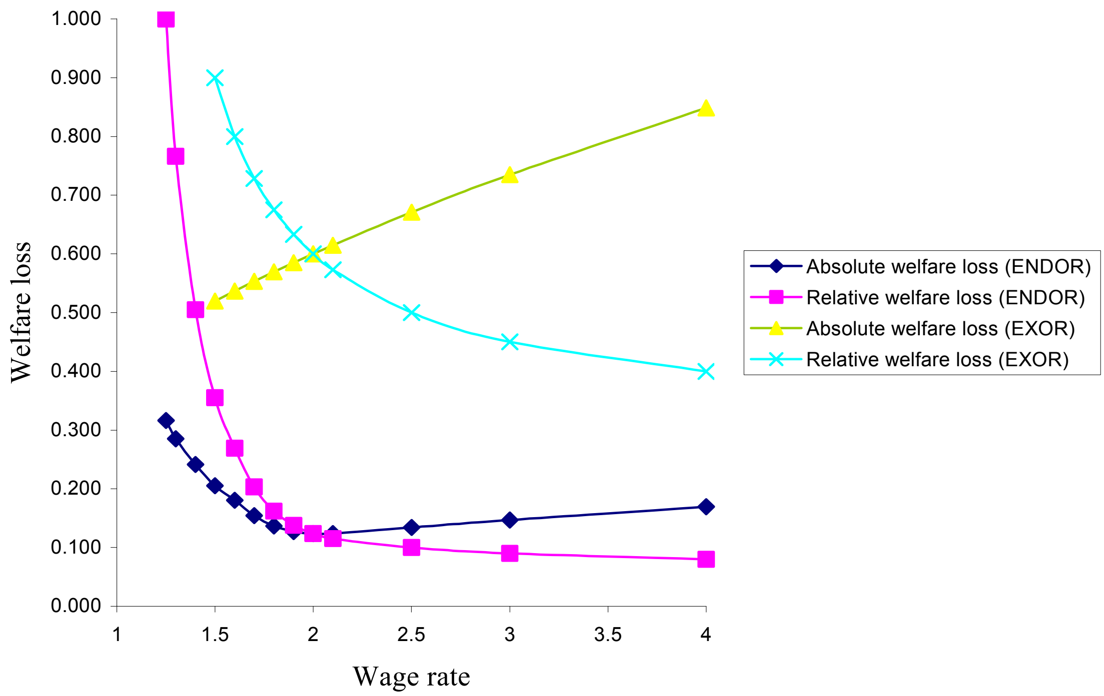


Figure 9: Welfare losses (absolute and relative) at various wage rates for illness case 2 and the EXOR model.



Note: The figures above are based on the following assumptions: a duration of the full recuperation, *r*, of 0.3, a weight on consumption in the utility function, α, of 0.5, and a net minimum consumption requirement, *a*, of 1. In addition we have assumed the productivity during illness, *s*, in the case 1 to be 0.5 and in the case 2 to be 0.8.

4. Limitations of a one-period model.

The model used here describes a time allocation between different activities within one period. Two aspects of the model are open to criticism: a) the (implicit) assumption of a utility discount rate of 0 for within-period time allocation; and b) the absence of inter-temporal linkages.[29] We will here provide some arguments in support of the relatively simple structure chosen, and provide indication on where extensions to the model might lead.

Regarding the first issue, it should be noted that the model developed here covers a relatively short time period in which the individual experiences a certain amount of illness. Given the limited-period time-frame, we can assume that the effect of discounting would be small and therefore can be disregarded. This said, if within-period utility discounting were introduced, this can be done either based on the assumption of a constant or an endogenous discount rate. It has been suggested that lower income groups, because of circumstance and environment, have lower ability to defer gratification (Maital and Maital, 1978). In addition, one could envision an extension of our model where e.g. the wage rate is endogenous and determined by prior investments in human capital, and therefore low-wage individuals would be those with higher utility discount rates.[30] Applied to our model, which could de seen as describing a world with two sub-periods, a lower wage rate could therefore be associated with a higher utility discount rate. The less the individual cares about tomorrow, the more he will be inclined to work today (during recuperation sub-period), as he will care less about recuperating in order to be healthy tomorrow, and more about obtaining some consumption also in the first sub-period. Hence, inclusion of time discounting could possibly reinforce the result that illness leads to increasing inequality.[31] A proper analysis of these issues, however, is beyond the scope of this paper, and would have to take into account the possible effect on the discount rate of the linkage between current labour supply and second period health.

The more compelling issue concerns the absence of inter-temporal linkages in our model. On the one hand, one can think of how the decision on the extent of recuperation today affects the health stock tomorrow and so, presumably, (i) the probability of falling ill tomorrow; and (ii) the ease of recuperating tomorrow conditional on falling ill. In particular, a realistic assumption would be that sickness is positively serially correlated, such that an increase in first period sickness due to the individual labour-choice raises the expectation of second period illness. This would lead to interesting contradictory effects of current sickness on current hours of work in a two-period framework; an increase in current period sickness has a reducing effect on current efficient labour supply whereas an expectation of future sickness has an increasing effect.[32]

On the other hand, a proper inter-temporal theory would also have to look at savings and labour supply decisions. By not returning to work today the individual will

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[29] Implicit utility discount rate of infinity between periods.

[30] On the other hand, people may decide to forego higher wages in the present by taking safer jobs that increase their chance of future survival, which would imply lower discount rates.

[31] Regarding endogenous determination of the discount rate, this would be driven by the linkage between current labour supply and second period health. In a paper by Becker and Mulligan (1997) the endogenous determination of time preferences is analysed. The authors note that individuals may alter their time preference in part by time and effort spent in forming mental pictures of future pleasures. They cite as examples of such time and effort activities like acquiring information through schooling, access to print media, and time spent with older persons, particularly parents. One could think of extensions to our model where time spent recuperating could simultaneously be spent on anticipation, which again would imply that higher-wage individuals have a lower discount rate.

[32] According to O'Donnell (1995) empirical evidence reveals a negative effect of sickness on labour supply, suggesting the direct effect of current illness outweighs the one through the expectation of future sickness.

accumulate less savings or, alternatively, more debt, which could intensify the pressure to return to work early tomorrow if one did fall ill. Furthermore, the decision not to return to work today could affect the probability of being laid off or getting employment tomorrow. Equally, if there is learning, the decision to stay at home and recuperate could lower productivity tomorrow. Hence there are many complex inter-temporal linkages between decisions today and stocks of health, financial assets, and human-capital assets. It is therefore not at all clear that introducing inter-temporal considerations would lead to greater pressures to recuperate.

Any proper analysis of these issues has to be infinite horizon. A finite horizon model – say a 2-period model - always has a last period. This last period is essentially a one period model just as the one presented in this paper and it is difficult to know just how sensitive the conclusions in the early period are to the fact that there is an ultimate final period. A proper infinite horizon model that also models the within-period recuperation decision, however, would be intractable analytically.

5. Conclusion and Policy Implications

The existing literature has so far given little attention to the time allocation choice an individual faces during sickness time, nor has the effect of that choice upon the duration of sickness time been analysed. This paper has attempted to cover this important gap.

Using standard utility maximisation theory we have analysed labour supply behaviour under constraints imposed by sickness and minimum consumption requirements. Departing from the previous literature, productivity in work and leisure is influenced by the labour supply decision in this model: if the individual works during exogenously imposed recuperation time, productivity in the post-recuperation time suffers, and this produces a non-linear efficient time budget constraint. The analysis finds that if any leisure is taken in the post-recuperation phase then recuperation is spent entirely in leisure (recuperating) - implying that the more leisure an individual takes, the more likely she is to stay at home during recuperation.

For a rather general utility function, an empirically supported elasticity of substitution between leisure and consumption, and under the assumption that no sick pay is received, we therefore find that only higher-wage individuals will choose to recuperate fully, whereas others will work while being sick as long as this yields additional efficient labour hours. Due to the illness characteristics, working during the full recuperation time in the severe illness case (case 1) does not yield additional efficient labour time, whereas in the less severe illness case (case 2) trade-off between efficient leisure and efficient labour is possible during the full recuperation period.

The main implication of this finding is that, under certain conditions, the income and welfare losses (relative and absolute) from illness are larger for low-wage groups than for those with higher wages. This, in turn, suggests that certain common illnesses or illness-reducing policies (for instance vaccination projects or pollution-reduction initiatives) may have income and welfare redistributing effects. The exogenous recuperation model (EXOR), in contrast, predicts absolute income and welfare losses to uniformly increase in wages and is hence seriously misleading if illness is in fact a time of choice. Furthermore, we have shown that commonly-used measurements of illness can lead to distorted estimates by neglecting the interaction between income and behavioural choices.

In order to appreciate the relevance of these findings, in particular for developing countries where real wages are low and sick-pay rarely part of the wage-package, we can turn briefly to published data on the percentage of the population in different countries below a nationally or internationally defined poverty line.[33] For example, in Honduras 68.8 per cent of the population have incomes below the international poverty line defined as $2 a day (2002 international prices, adjusted for purchasing power parity), and for the Latin American and Caribbean regions overall the percentage is 35. If we think of the poverty line as being slightly higher than the minimum consumption requirement in our model, some interesting insights ensue. Recall that a wage rate of 1 is necessary in order for a healthy individual to exactly fulfil the net minimum consumption requirement when working full time, and that under illness this survival wage rate is higher than 1 (1.25 in the less severe illness case, and 1.43 in the more severe illness case). Let us for simplicity suggest that somewhere below a wage rate of 2 lies the poverty line (the exact location depends on the sickness specifications) in the model that has been presented. This implies that in a country like Honduras, the majority of the population falls within the range of wage rates within which both absolute and relative welfare losses - and to a lesser degree absolute and relative income losses - may be decreasing in wages, i.e. when moving from those individuals with the lowest wage rates within this group to those with higher wage rates, the losses incurred from a certain illness may be decreasing. A health shock to the Honduran population, such as a major SARS epidemic, could hence lead to a worsened income distribution in the country, and an environmental policy that decreases the amount of particulate matter in the air (and thereby mortality and morbidity from air pollution) or a vaccination campaign could have income distributional implications.

The model can be used to analyse the merits of sick pay policy. We will limit our discussion here to the issues of full sick pay and poor-only sick pay, and will assume that moral hazard is not an issue, and that it is not possible to obtain any sick pay for time spent working during part of the recuperation period. Under the concept of ‘full sick pay’ we understand receiving full compensation while being off sick for those hours one would usually work when healthy. Under the concept ‘poor-only sick pay’ we mean that sick pay (in this case full sick pay) is only given to those who have such a low wage rate that, given the net minimum consumption requirement on their wages, they would choose to work during recuperation if that implied more efficient working time (i.e. they have a wage rate at or below the critical wage in our model). With full sick pay, everybody will recover fully and even the poorest individuals who barely survive even under healthy conditions (and who would not survive without sick pay when falling ill) will now also survive under illness. With poor-only sick pay, the same will be true, since all the non-poor choose to recuperate fully anyway even without sick pay. No income losses are therefore incurred with full sick pay, whereas only the non-poor will incur income losses when sick pay is restricted to the poor. For the non-poor, income losses will increase with their wages.

Interestingly, with full sick pay the low-wage individuals will be better off (in terms of welfare) when they are sick than when they are healthy, whereas a high-wage individual who cares mostly about leisure would still suffer welfare losses from illness. The reason for this perverse effect of illness for the poor is that they spend most of their time working when they are healthy. With sick pay, they would continue receiving the full-productivity wage as handouts, while being in fact at home recuperating, from which they would draw some leisure utility. On the other hand, high-wage individuals with a high preference for leisure activities will still suffer under illness because they get less utility out of their leisure time when ill. Nevertheless, they would be better off with than without sick pay.[34]

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[33] World Bank (2002).

[34] This analysis is partial and disregards how sick pay is paid for.

At an amount of sick pay less than the full amount, the low-wage individuals will have to trade off the amount they would receive staying home with the one they could earn while continuing working, taking into account the fact that they may actually work more hours during the sickness period than they would have had they been healthy during the same time-period. Depending on the size of the parameters it may still be an optimal choice for the poorest to work during the recuperation time.

The next question is whether it would be beneficial for firms to pay the sick. If the firm pays the sick in our model, it basically has the same labor cost as under healthy conditions but for less production. The intuition here is quite clear: in our model, wage is paid according to efficiency and any person who chooses to work during illness does so to increase his or her production and hence income. Only in the case of the lowest-wage individuals, i.e. those who would otherwise not survive, would sick pay lead to higher production, but by paying these workers sick pay the firm would be paying them more than their marginal product. A small, price-taking, profit-maximising firm would therefore not choose to pay sick pay in the current model.

A proper analysis of benefit packages and contracts, especially in an inter-temporal framework, would have to take into account a host of other issues such as how firms go about attracting their preferred types of employees, the psychological effect on production of health insurance, the uncertainty over sickness developments if employees do not recover fully from their illness, training costs if additional staff has to be taken on, internalisation of production externalities if sickness is related to the workplace etc.

Regarding society, as opposed to the firm, we can assume it is maximising a social welfare function. As we have seen in our analysis of absolute welfare and income losses without sick pay, it is possible that the lowest-wage individuals may incur larger losses than the higher-wage individuals from a certain illness. With full sick pay for all no income losses are incurred, however the low-wage individuals may in fact experience a welfare gain from falling sick under this compensation-scheme, whereas the higher-wage individuals may still suffer. Although a proper analysis is required to understand the welfare effects of introducing sick pay, taking both the welfare effects on different wage-groups and their willingness or ability to pay compensation into account, there seems to exist the possibility of potential Pareto improvement, whether it is a compensation for all or a poor-only sick pay policy.

Nevertheless, paying sick pay to everybody, irrespective of the wage rate, will lead to a deadweight loss because the high-wage individuals were going to recuperate anyway. The deadweight loss is here related to the fact that with full sick pay, individuals can substitute some of the work in the post recuperation period for leisure and hence productivity and output will decrease. In the severe illness case, sick pay will allow some of those low-wage individuals who could not survive without the policy to actually survive. For less severe illnesses, sick pay will prevent them from working during illness.

In addition to the above-mentioned deadweight loss, losses could result from the inefficiencies inherent in operating a sick pay program: e.g. administrative costs, the cost of moral hazard through increased reported illness in response to the existence of this program, and any deadweight loss from financing these benefit payments.

Finally, we will try to answer a normative question, namely; would it be desirable to have sick pay? We have seen that income and welfare losses from a given illness may be larger for the low-wage individuals (including death in the case of the very poorest) than for those with higher wages, and hence having serious equity effects. For this reason, sick pay would clearly be a redistributive policy increasing equity, especially when paid only to the low-wage individuals, and may be desirable for this reason. A more compelling argument for centrally provided sick pay or public intervention is inefficiency or simple absence of credit and insurance markets, which prevents an individual to protect against dire effects of illness.

Another interesting theoretical extension of the model would be to consider the joint determination of recuperation time and other health investments (e.g. medical expenditures, obtaining medical treatment). Finally, the relevance and importance of the insights from the model developed in this paper will become clearer once the model is tested empirically.

Appendix A. Hybrid case

The hybrid case, case 3, is that where less than full recuperation, i.e. $0 < x < 1$, yields the maximum amount of obtainable efficient labour. From the Kuhn-Tucker conditions above we find that $0 < x < 1$ implies $\mu = 0$ and $[\partial \hat{\Phi} / \partial x] = 0$. This then yields the following condition for case 3:

$$\sigma'(x) = \frac{sr}{1-r} \tag{A.1}$$

Substituting in the highest and lowest possible amounts of actual recuperation, i.e. $x=1$ and $x=0$, and taking into account the fact that we have assumed decreasing returns to recuperation, case 3 then has to fulfil the following condition:

$$\sigma'(0) > \frac{sr}{1-r} > \sigma'(1) \tag{A.2}$$

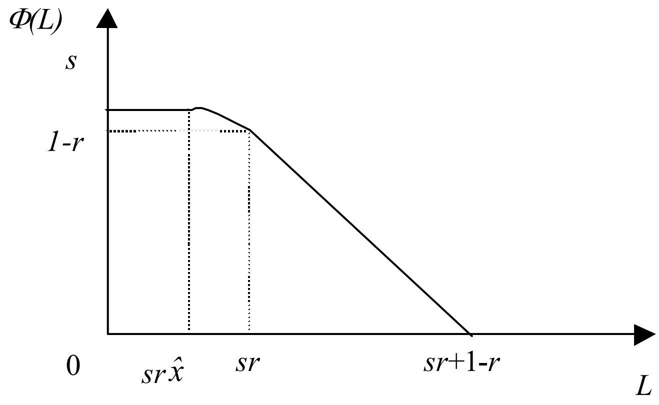
This case implies that there is some amount of actual recuperation between zero and one at which the marginal productivity in the post-recuperation phase of an additional unit of actual recuperation multiplied by the length of the post-recuperation period, is equal to the marginal productivity of an additional time unit spent working during the recuperation phase multiplied by its length. Then there exists an x, $\hat{x}$, $0 < \hat{x} < 1$, so that $\sigma'(\hat{x}) = sr/(1-r)$, and $\hat{\Phi} = sr(1-\hat{x}) + \sigma(\hat{x})(1-r) > 1-r$. Then efficient labour is given by the following two expressions:

$$\Phi(L) = \begin{cases} \hat{\Phi}, 0 \le L \le sr\hat{x} \\ sr - L + (1-r)\sigma\left(\dfrac{L}{sr}\right), sr\hat{x} \le L \le sr \end{cases}$$

Figure 10 depicts the time constraint under case 3. The maximum amount of leisure available is the same as in case 1: $L = s\ r + 1\text{-}r$. However, a certain amount of efficient leisure during the recuperation time, $sr(1-\hat{x})$, can now be traded against additional work time, with a positive effect on overall efficient work time.

Below the point marked $s\ r\hat{x}$ on the leisure axis there is no more efficient work to be gained by trade, because any additional work during recuperation ($>1-\hat{x}$) will be at least offset by the ensuing decrease in post-recuperation productivity at work.

Figure 10: Efficient time opportunity frontier in the case where working part of the recuperation time yields the maximum amount of efficient labour (case 3).



Appendix B.1. Additively and non-additively separable utility functions

Additively separable utility function.

Utility, a function of consumption and leisure, is maximised subject to the budget constraint:

$$\max_{0\le L\le 1} U\left[w\Phi(L)-a, L\right] \tag{B.1}$$

This yields the familiar first order condition:

$$U_C w\Phi'(L) + U_L \le 0 \tag{B.2}$$

Following the envelope theorem, the marginal effect of wage upon leisure, i.e. the sign of *dL(w)/dw*, has the same sign as the following expression:[35]

$$\frac{\partial}{\partial w}\left[U_c\left(w\Phi(L)-a, L\right)w\Phi'(L) + U_L\left(w\Phi(L)-a, L\right)\right]$$

Let us assume an additively separable utility function, i.e.: $U = f(c) + g(L)$. Then, the sign of *dL(w)/dw* will be equal to the sign of the following expression:[36]

$$-\left\{1-\frac{w\Phi(L)}{(w\Phi(L)-a)}\left(-\frac{cf''}{f'}\right)\right\} \tag{B.3}$$

Let us define $\lambda = \left(-cf''/f'\right)$. The lambda, λ, is thus the elasticity of the marginal utility of consumption with respect to consumption, also known as the reciprocal of the elasticity of substitution, σ. Expression (B.3) is hence equivalent to equation (13) in the paper.

Perfect Substitutes.

First, assume consumption and leisure to be *perfect substitutes*, e.g.:

$$U(c, L)=\alpha(w\Phi(L)-a)+\beta L$$

Now what we want is the sign of $dL(w)/dw$, which will have the same sign as the expression:

$$\frac{\partial}{\partial w}\left[\alpha w\Phi'(L)+\beta\right] = \alpha\Phi'(L)<0$$

We know from before that $\Phi'(L) < 0$ and hence the sign of $dL(w)/dw$ is negative.

Perfect Complements.

Next we let consumption and leisure be *perfect complements*, e.g.:

[35] See appendix B.2. below for mathematical explanation.

[36] For calculations see the appendix B.3.

$U\ (c, L) = min\ (\alpha\ (\ w\ \Phi\ (L) - a),\ \beta L).$

In this case the derivative of the first order condition with respect to wage is the following:

a) For $\alpha\ (\ w\ \Phi(L) - a)$ being the minimum of $min\ (\alpha\ (\ w\ \Phi\ (L) - a),\ \beta L):\ \alpha\ \Phi'\ (L)$

b) For βL being the minimum of $min\ (\alpha\ (\ w\ \Phi\ (L) - a),\ \beta L):\ =0$

The sign of $dL(w)/dw$ in case a) is negative, whereas in case b) wage has no effect on leisure.

Quasilinear Preferences.

Let us now look at *quasilinear* preferences between consumption and leisure, e.g.:

$U\ (c, L) = \sqrt{(w\Phi(L)-a)} + L$

In this case the derivative of the first order condition with respect to wage is:

$$\frac{\partial}{\partial w}\left[\frac{1}{2}\left(w\Phi(L)-a\right)^{-\frac{1}{2}}w\Phi'(L)+1\right]=$$

$$-\frac{1}{4}\left(w\Phi(L)-a\right)^{-\frac{3}{2}}\Phi(L)w\Phi'(L)+\frac{1}{2}\left(w\Phi(L)-a\right)^{-\frac{1}{2}}\Phi'(L)=$$

$$\frac{1}{2}\left(w\Phi(L)-a\right)^{-\frac{1}{2}}\Phi'(L)\left[1-\frac{\Phi(L)w}{2\left(w\Phi(L)-a\right)}\right]$$

The first part of the expression above (before the term in brackets) is negative as long as consumption is positive. Then $dL(w)/dw$ will be of a positive sign if $\frac{a}{w} > \frac{\Phi(L)}{2}$, and of a negative sign otherwise.

For the opposite quasi-linear indifference curve, e.g. $U(c,L)=\sqrt{L}+(w\Phi(L)-a)$, we have:

$$\frac{\partial}{\partial w}\left[\frac{1}{2}L^{-\frac{1}{2}}+w\Phi'(L)\right]=\Phi'(L)$$

Hence $dL(w)/dw$ is negative.

As we see the results are very sensitive to the type of utility function we use. It seems, however, unlikely that consumption and leisure should be perfect substitutes or perfect compliments. In that case, $\frac{dL(w)}{dw}$ can take both a positive and a negative sign.

Appendix B.2. Envelope Theorem

We know that the optimal choice function *L (w)* must satisfy the condition $\frac{\partial f}{\partial L}\left[L(w), w\right] \equiv 0$. Differentiating both sides of this identity gives:

$$\frac{\partial^2 f\left(L(w),w\right)}{\partial L^2}\frac{dL(w)}{dw}+\frac{\partial^2 f\left(L(w),w\right)}{\partial L\partial w}\equiv 0$$

Solving for *d L (w) / d w*, we have

$$\frac{dL(w)}{dw}=-\frac{\dfrac{\partial^2 f\left(L(w),w\right)}{\partial L\partial w}}{\dfrac{\partial^2 f\left(L(w),w\right)}{\partial L^2}}$$

The denominator of this expression is negative due to the second-order condition for maximisation. Noting the minus sign preceding the fraction, we can conclude that the sign of $\frac{dL(w)}{dw}$ equals the sign of $\frac{\partial^2 f\left(L(w), w\right)}{\partial L \partial w}$.

Appendix B.3. Derivation of Expression (13)

$$\frac{\partial}{\partial w}\left[U_c\left(w\Phi(L)-a, L\right)\right]w\Phi'(L) = U_c(w\Phi(L)-a, L)\Phi'(L) + U_{cc}(w\Phi(L)-a, L)\Phi(L)w\Phi'(L)$$

$$= U_c(w\Phi(L)-a, L)\Phi'(L)\left[1+\frac{U_{cc}\Phi(L)w}{U_c}\right] = U_c(w\Phi(L)-a, L)\Phi'(L)\left[1-\left(\frac{\Phi(L)w}{\Phi(L)w-a}\right)\left[-\frac{cU_{cc}}{U_c}\right]\right]$$

(Note: $c = \Phi(L)w - a$). Since U_c is positive and $\Phi'(L)$ is negative, the expression above will be of the same sign as expression (13) in the text, i.e.:

$$-\left[1-\left(\frac{\Phi(L)w}{\Phi(L)w-a}\right)\left(-\frac{cU_{cc}}{U_c}\right)\right]$$

Appendix C. Derivation of wages

Critical Wage Derived in the Cases Where σ = 1 and σ = 0.5

Analytical expressions for the critical wage in the Cobb-Douglas case where $\sigma = 1$:

Assuming that $\overline{L} \leq T/(1+\gamma)$, the critical wage when $\sigma = 1$ can be expressed as follows:

$$\overline{w} = \frac{a}{T-(1+\gamma)\overline{L}} = \frac{a}{1-r-sr\frac{\alpha}{1-\alpha}}$$

with

$$\frac{\partial\overline{w}}{\partial a} = \frac{1}{T-(1+\gamma)\overline{L}} > 0, \frac{\partial\overline{w}}{\partial\alpha} = \frac{a\overline{L}}{\left(1-r-\overline{L}\frac{\alpha}{1-\alpha}\right)^2(1-\alpha)^2} > 0$$

$$\frac{\partial\overline{w}}{\partial s} = \frac{ar\frac{\alpha}{1-\alpha}}{\left(1-r-\overline{L}\frac{\alpha}{1-\alpha}\right)^2} > 0, \frac{\partial\overline{w}}{\partial r} = \frac{a\left(1+s\frac{\alpha}{1-\alpha}\right)}{\left(1-r-\overline{L}\frac{\alpha}{1-\alpha}\right)^2} > 0$$

The critical wage $\overline{w}$ is strictly increasing in *a,* and thus strictly decreasing in unearned income, $\overline{y}$. The critical wage is further increased with sickness duration, *r*, productivity during sickness, *s*, as well as with the weights given to consumption in the preference function, α.

Wage range in illness case 1.

We can calculate the range of wages for which individuals would choose to exactly recuperate for the first illness case, i.e. $\sigma'(1) = \beta\,(1 - s^{1/\beta}) \geq s\ r\ /(1-r)$. It was shown that for this sickness case the total effect on efficient working time from working during illness would not be positive, and hence no one would work during recuperation. The existence wage is then given by the net minimum consumption requirement divided by the maximum obtainable efficient labour time:

$$w_{min} = a\ /(1-r) \leq w$$

Hence, at any wage between the existence wage, w_{min}, and the critical wage, $\overline{w}$, the individuals will exactly recuperate.

Wage range in illness case 2.

In the second illness case, where $\frac{sr}{1-r} \geq \sigma'(0) = \beta s^{(\beta-1)/\beta}(1-s^{1/\beta})$, we found the following expressions for maximum working time for given amounts of leisure:

$$\Phi(L) = sr - L + (1-r)\sigma\left(\frac{L}{sr}\right) \qquad \text{for } 0 \leq L \leq s\,r \qquad (c.1)$$

$$\Phi(L) = 1 - r \qquad \text{for } L = s\,r \qquad (c.2)$$

Substituting for $x=0$ in equation (c.1) gives us the following existence wage:

$$w_{min} = a / s \leq w$$

Hence, at any wage between the existence wage and the critical wage individuals will choose an amount of efficient leisure between $0 \leq L \leq s\ r$. The exact amount can only be derived numerically after having assumed different parameter values.

For those who choose an amount of leisure locating at the kink (i.e. expression (c.2) we can substitute for x=1 in equation (d.6) (below) and the wage that equalises the two sides gives the lower kink-wage, $\breve{w}$. The upper kink-wage would be the critical wage discussed earlier.

$$\frac{\alpha\breve{w}}{1-\alpha}\left(1 - \frac{1-r}{sr}\sigma'(1)\right) = \left[\frac{\breve{w}(1-r)-a}{sr}\right]^{\lambda}$$

Appendix D: Steps to arrive at income loss measurements

Regime 1

Let us define total "choice" time, T_C, as the amount of total time over and above what is required to fulfil the net minimum consumption requirement (i.e. $T_C = T - a / w$, where a is the net minimum consumption requirement, and when divided by the wage rate, w, gives the amount of labour hours required to fulfil the requirement). Total time was assumed equal to one in the healthy case ($T^H = 1$), and in the regime 1 it was found to be given by the following expression: $T_1^S = 1 - r(1-s)$. Superscripts H and S indicate the healthy and sick scenario, respectively, and subscript 1 indicates regime 1.

In section 2.2 (equation (14)) we derived the utility maximising choice of leisure for the CES case and by substituting in $\sigma = 1$ we also have the C-D case. Leisure in the healthy case and in the case of illness for those individuals who take out leisure in post-recuperation time (i.e. $L \geq s\ r$) is then given by the following expression in the C-D case:

$$L = (1-\alpha)(T_C) \qquad (d.1)$$

Notice that if the healthy individual has a minimum consumption requirement on wages that is equal to the income he earns when working full time then the individual will not spend any time in leisure. The weighting of the "choice" time in terms of leisure and labour in the C-D case are given by $1\text{-}\alpha$ and α, respectively.

Based on equations (d.1), and (5), income in the healthy case and the regime 1 illness case is given by the following expression:

$$y = \alpha w(T_C) + a \qquad (d.2)$$

The amount of income earned is therefore the sum of the necessary amount needed to fulfil the minimum consumption requirement, a, and the utility-weighted additional possible amount to be earned.

Substituting total choice time in the healthy scenario and in the regime 1 illness scenario into equation (d.2), and consequently subtracting the latter from the former yields the expression for absolute income loss (AIL) – given in expression (18) in the paper. Dividing the AIL by healthy income gives us the expression for the relative income loss (RIL):

$$RIL_1 = \frac{\alpha wr(1-s)}{\alpha(w-a)+a} \qquad \text{with} \qquad \frac{\partial RIL}{\partial w} = \frac{r(1-s)a\alpha(1-\alpha)}{(\alpha(w-a)+a)^2} > 0 \qquad (18')$$

Illness case 1

In case 1 we found that the maximum amount of labour hours was achieved when exactly recuperating, i.e.:

$\Phi(L)_{2,1} = 1 - r$ for any $0 \leq L \leq s\, r$ *(d.3)*

Subscript 2,1 indicates that we are looking at low-wage individuals (i.e. regime 2) for illness case 1. Multiplying efficient labour (equation (d.3)) by the wage rate, income takes the form: $y_{2,1}^{S} = w(1-r)$

The expressions for absolute and relative income loss can then be derived based on equations (d.2), and (d.3), the former is given in expression (19), whereas RIL is given by dividing the AIL by healthy income:

$$RIL_{2,1} = \frac{\alpha(w-a)+a-w(1-r)}{\alpha(w-a)+a} \quad \text{with} \quad \frac{\partial RIL}{\partial w} = -\frac{(1-r)(1-\alpha)a}{(\alpha w+(1-\alpha)a)^2} < 0 \qquad (19')$$

Illness case 2

For the cases where the individual chooses an amount of actual recuperation time less than full recuperation, an assumption on the functional form of post-recuperation productivity is necessary. Let us hence assume the following: $\sigma(x)=(x+ (1-x)\, s^{1/\beta})^{\beta}$. This expression fulfils the conditions we assumed initially, i.e. $\sigma(0)=s;\ \sigma(1)=1,\ s \leq \sigma(x) \leq 1$ when $0 \leq x \leq 1$, and gives us positive and diminishing returns to recuperation time, i.e.:

$$\sigma'(x) = \beta\left(x+(1-x)s^{1/\beta}\right)^{\beta-1}\left(1-s^{1/\beta}\right) > 0$$
$$\sigma''(x) = \beta\left(1-s^{1/\beta}\right)^2(\beta-1)\left(x+(1-x)s^{1/\beta}\right)^{\beta-2} < 0$$

With these specifications the optimal amount of leisure and work can be calculated for different income groups and for the healthy and sick case. The absolute income loss from illness is calculated by multiplying the wage rate with the amount of efficient hours worked in the healthy and sick case, and then subtracting the latter from the former. Relative income loss is the ratio of the absolute income loss to the income earned in either the healthy or the sick case. We have chosen to use the healthy income for this purpose.

The expressions for maximum working time for given amounts of leisure were derived in section 2:

$$\Phi(L) = sr - L + (1-r)\sigma\left(\frac{L}{sr}\right) \quad \text{for } 0 \leq L \leq s\, r \qquad (d.4)$$

$$\Phi(L) = 1 - r \quad \text{for } L = s\, r \qquad (d.5)$$

Let us first analyse the case where $0 \leq L \leq s\ r$. Maximising utility with respect to the budget constraint gives the following first order conditions:[37]

$$L = \frac{1-\alpha}{(1-\alpha)w-\alpha}\left(w\left(sr+\sigma\left(\frac{L}{sr}\right)(1-r)\right)-a\right) \qquad (d.6)$$

Since leisure is equal to the amount of the recuperation time (r) that is spent recuperating (x) multiplied by the efficiency units of leisure during recuperation time (s) (i.e. $L = srx$), we can substitute this expression for leisure into equation (d.6) above, and thereby solve the equation for x by the use of simulations. The resulting value of x can then be multiplied by the duration of the full recuperation time, and the efficiency units of leisure during recuperation time, to give the amount of leisure chosen.

Income under sickness now takes the form:

$$y_{2,2}^{S} = w\left(sr(1-x)+\sigma(x)(1-r)\right) \qquad (d.7)$$

This expression for income is a function of the productivity in the post-recuperation time, which is a function of the amount of actual recuperation, which in turn is a function of the wage rate. We can therefore only derive exact expressions for the absolute and relative income losses for the individuals with wages

[37] The maximisation problem is:

$$\underset{L}{Max}\left(w\left(sr-L+\sigma\left(\frac{L}{sr}\right)(1-r)\right)-a\right)^{\alpha} L^{1-\alpha}$$

within the boundaries of this wage group numerically. The generic AIL expression is given in expression (20), whereas for RIL it is the following:

$$RIL_{2,2}=\frac{w\alpha-\alpha a+a-w\left(sr-L+(1-r)\sigma\left(\frac{L}{sr}\right)\right)}{w\alpha-\alpha a+a} \qquad (20')$$

The expressions for absolute and relative income loss for the low-wage individuals in case 2, regime 2, (indicated by the subscript 2,2 below) can be derived based on equations (d.2) and (d.7), in the special cases of $x=0$ and $x=1$:

$$AIL_{2,2}=\begin{cases}\alpha(w-a)+a-ws, x=0\\ \alpha(w-a)+a-w(1-r), x=1\end{cases} \qquad (d.8)$$

$$RIL_{2,2}=\begin{cases}\frac{\alpha(w-a)+a-ws}{\alpha(w-a)+a}, x=0\\ \frac{\alpha(w-a)+a-w(1-r)}{\alpha(w-a)+a}, x=1\end{cases} \qquad (d.9)$$

By substituting for $x = 0$ and x=1 in equation (d.6) and finding the wage that makes the two sides of the equation equal we find the range within which the wage rate will lie:

$$\frac{a}{s}\leq w<\frac{a(1-\alpha)}{(1-r)(1-\alpha+\alpha\sigma'(1))-\alpha sr} \qquad (d.10)$$

Substituting the wage boundaries into the expressions (d.8) and (d.9), the following boundaries for absolute and relative income losses result:

$$AIL_{2,2}=\begin{cases}\alpha a\left(\frac{1}{s}-1\right), x=0\\ \alpha a\left(\frac{r(1-s)+\sigma'(1)(1-r)}{(1-r)(1+\gamma\sigma'(1))-\gamma sr}\right), x=1\end{cases} \qquad (d.11)$$

$$RIL_{2,2}=\begin{cases}1-\frac{s}{\alpha+s(1-\alpha)}, x=0\\ 1-\frac{1-r}{\alpha(1-sr)+(1-r)(1-\alpha+\alpha\sigma'(1))}, x=1\end{cases} \qquad (d.12)$$

Comparing the expressions for AIL when wage is low (here: $x = 0$) and wage is higher ($x=1$) given in expression (d.11) and (d.12) we find that the AIL and RIL are lower at the higher wage if the following conditions hold:

$$\frac{s}{1-r}<\left[\frac{(1-s)(1-\alpha+\alpha\sigma'(1))}{r(1-s)+\sigma'(1)(1-r)(1-\alpha)}\right] \qquad (d.13)$$

The RIL is lower at the higher wage if the following condition is fulfilled:

$$\frac{s}{1-r}<\left[\frac{1}{1-sr+(1-r)\sigma'(1)}\right] \qquad (d.14)$$

Keeping in mind that for illness case 2 the productivity during illness, *s*, is larger than the duration of the post-recuperation period, *1-r*, these conditions imply that the numerators on the right hand side (RHS) are larger than the denominators.

Appendix E: Steps to arrive at comparable welfare loss measurements

Regime 1

Indirect utility is found by substituting the utility maximising choice of leisure (equation (14)) into the budget constraint (equation (22))) solving for consumption and substituting the two Marshallian demand functions for consumption and leisure back into the utility function:

$$V^k = v\left[w, T^k w - a\right] = \left(F^k\right) w^{-(1-\alpha)} \alpha^{\alpha} (1-\alpha)^{1-\alpha}, \qquad \frac{\partial v}{\partial w} > 0 \qquad (E.1)$$

Full income, F^k, was given by the following expression in the healthy case: $F^H = w - a$. In the sickness scenario regime 1 we had the following expression for full income: $F^H = w(1 - r(1-s)) - a$. Solving equation (E.1) for full income, we can then express the income equivalent of utility at a reference wage, $\overline{w}$, in the following manner:

$$F^k_{\overline{w}} = \frac{v(w)\overline{w}^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \qquad (E.2)$$

where $F^k_{\overline{w}}$ indicates the minimum amount of money in excess of the net minimum consumption requirement required at a reference wage ($\overline{w}$) to support the level of indirect utility v^k.

By substituting the expressions for full income in the healthy and sick case into the indirect utility expressions given in equation (E.1), and substituting these again into the expressions for full income equivalents of welfare at a reference wage (equation (E.2)), and then finally substituting the latter into the general expressions for absolute and relative welfare losses (both related to expression (21)), we obtain the expression for absolute comparable welfare loss (ACWL) given in equation (23). Differentiating it with respect to wages yields:

$$\frac{\partial ACWL}{\partial w} = \left(\frac{\overline{w}}{w}\right)^{1-\alpha}\left[\alpha + \frac{a}{w}(1-\alpha) - \left(\frac{wsr\alpha}{\left(w(1-r)-a\right)(1-\alpha)}\right)^{1-\alpha}(1-r)\right] = ?$$

Relative comparable welfare loss (RCWL) is obtained by dividing ACWL by the full healthy income equivalent:

$$RCWL_1 = \frac{wr(1-s)}{w-a} \qquad \text{with } \frac{\partial RCWL}{\partial w} < 0 \qquad (23')$$

Illness case 1

It was previously shown that individuals with wages below the critical wage, $\widetilde{w}$, in illness case 1 would locate their choice of leisure time exactly at the kink, where $L = s\,r$ and $H = 1 - r$. Their indirect utility is therefore:

$$v^k_{2,1} = \left(w(1-r) - a\right)^{\alpha}\left(sr\right)^{1-\alpha} \qquad \frac{\partial v}{\partial w} > 0 \qquad (E.3)$$

The full-income equivalent measure of welfare using a reference wage rate was given in equation (E.2). Substituting the latter into the general expressions for absolute and relative welfare losses (both related to expression (21)), we obtain the expression for absolute comparable welfare loss (ACWL) given in equation (24). Relative comparable welfare loss (RCWL) is obtained by dividing ACWL by the full healthy income equivalent:

$$RCWL = 1 - \frac{\left(1 - r - \frac{a}{w}\right)^{\alpha}(sr)^{1-\alpha}}{(1-\frac{a}{w})\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}} \qquad \text{with } \frac{\partial RCWL}{\partial w} < 0 \qquad (24')$$

Illness case 2

For the individuals who choose a bundle located on the curved part of the budget constraint we know that at the points of tangency between the efficient time budget constraint and the indifference curve the slope of the budget constraint is equal to the marginal rate of substitution between efficient labour and efficient leisure (negative of the ratio of the marginal utility of leisure to the marginal utility of labour), i.e.:

$$\Phi'(L) = -\frac{\partial U / \partial L}{\partial U / \partial H} = -\frac{1-\alpha}{w\alpha}\frac{C}{L} \tag{E.4}$$

From the third FOC related to the maximisation in expression (3), we obtain the following expression for leisure: $L = s\,r\,x < s\,r$. By substituting the ensuing expression for actual recuperation, i.e. $x = L / sr$, into the labour expression (4) we derive the expression for efficient labour:

$$\Phi(L) = s\,r - L + (1-r)\sigma\left(\frac{L}{sr}\right)$$

Differentiating this expression with respect to leisure will give us the slope of the efficient time budget constraint:

$$\Phi'(L) = \frac{1-r}{sr}\sigma'\left(\frac{L}{sr}\right) - 1 = -\lambda < 0 \tag{E.5}$$

By solving equation (E.4) for leisure, substituting the expression obtained into the budget constraint, and solving for consumption we obtain:

$$C = \frac{\alpha(-\Phi')(wT^S - a)}{1-\alpha-\alpha\Phi'} \tag{E.6}$$

Furthermore, substituting this expression for consumption into the expression for leisure we obtain:

$$L = \frac{(1-\alpha)(wT^S - a)}{w(1-\alpha-\alpha\Phi')} \tag{E.7}$$

We can now calculate the indirect utility, $v(w)$, by substituting these two Marshallian demand functions for consumption and leisure back into the utility function:

$$v(w) = \frac{(wT^S - a)\alpha^{\alpha}(1-\alpha)^{1-\alpha}(-\Phi')^{\alpha}}{(1-\alpha-\alpha\Phi')w^{1-\alpha}} \tag{E.8}$$

The absolute comparable welfare loss (ACWL) is then given in equation (25). Relative comparable welfare loss (RCWL) is obtained by dividing ACWL by the full healthy income equivalent:

$$RCWL = 1 - \frac{(wT^S - a)(-\Phi')^{\alpha}}{(w-a)(1-\alpha-\alpha\Phi')} \tag{25'}$$

The expressions for absolute and relative welfare loss for the low-wage individuals in case 2, regime 2, (indicated by the subscript 2,2 below) can be derived based on equations (d.10), (25) and (25') in the special cases of $x=0$ and $x=1$:

$$ACWL_{2,2} = \begin{cases} \dfrac{\overline{w}^{1-\alpha}a^{\alpha}(1-s)}{s^{\alpha}}, x=0 \\ \dfrac{\overline{w}^{1-\alpha}a^{\alpha}\left(r(1-\alpha(1-s)) - \alpha(1-r)\sigma'(1) - (sr-(1-r)\sigma'(1))^{\alpha}(sr)^{1-\alpha}\right)}{((1-r)(1-\alpha+\alpha\sigma'(1)) - \alpha sr)^{\alpha}(1-\alpha)^{1-\alpha}}, x=1 \end{cases} \tag{E.9}$$

$$RCWL_{2,2} = \begin{cases} 1, x=0 \\ 1 - \dfrac{(sr)^{1-\alpha}(sr-\sigma'(1)(1-r))^{\alpha}}{r(1-\alpha(1-s)) - (1-r)\alpha\sigma'(1)}, x=1 \end{cases} \tag{E.10}$$

Comparing the expressions for ACWL when wage is low (here: $x = 0$) and wage is higher ($x=1$), we find that the ACWL decreases when moving from the lower to the higher wage if the following condition holds:

$$\frac{1-s}{s^{\alpha}} > \frac{\left(r(1-\alpha(1-s))-\alpha(1-r)\sigma'(1)-\left(sr-(1-r)\sigma'(1)\right)^{\alpha}\left(sr\right)^{1-\alpha}\right)}{\left((1-r)\left(1-\alpha+\alpha\sigma'(1)\right)-\alpha sr\right)^{\alpha}(1-\alpha)^{1-\alpha}}$$

The relative welfare loss decreases when moving from the lower to the higher wage if the following condition is fulfilled:

$$1 > 1-\frac{\left(sr\right)^{1-\alpha}\left(sr-\sigma'(1)(1-r)\right)^{\alpha}}{r(1-\alpha(1-s))-(1-r)\alpha\sigma'(1)}$$

This condition is fulfilled in illness case 2. The numerator of the fraction on the RHS is positive due to the assumption defining this illness case, i.e.: $sr/(1-r)\geq\sigma'(0)\geq\sigma'(1)$, and the denominator is also positive (an alternative way of writing the denominator is the following: $\frac{w-a}{a}\left((1-r)\left(1-\alpha+\alpha\sigma'(1)\right)-\alpha sr\right)$).

Locating at the kink, where $L = s\, r$, will only be chosen when wage lies above the lower-bound kink-wage, $\breve{w}$, and below the critical wage. The analysis is the same as under illness case 1, only the lower boundary wage rate differs. In the Cobb-Douglas case the absolute welfare losses decrease with increasing wages if the ratio of labour in the healthy scenario to labour in the sickness scenario is smaller than the marginal rate of substitution between efficient leisure and efficient labour in the sickness scenario to the power of ($1-\alpha$), i.e. the weight given to leisure in the utility function, i.e.:

$$\frac{\alpha\left(w-a\right)+a}{w(1-r)} < \left(\frac{\alpha wsr}{(1-\alpha)\left(w(1-r)-a\right)}\right)^{1-\alpha}$$

We know from the analysis in illness case 1 that this condition is not fulfilled at the critical wage. When substituting in the lower boundary kink-wage given, we find that the condition will hold if the following inequality holds:

$$\left(1+\alpha\sigma'(1)+\frac{r\alpha(1-s)}{1-r}\right) < \left(\frac{sr}{sr-(1-r)\sigma'(1)}\right)^{1-\alpha}$$

Relative welfare loss was found to be decreasing with increasing wages.

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